Dedicamos este minissimpósio à memória do Prof. Jim Douglas Jr.

Programa do Minissimpósio

O minisimpósio é composto de duas sessões com 5 palestras, sendo cada palestra com duração de 25 minutos.

Primeira Sessão (Coordenadores: Abimael Loula – LNCC e Cristiane Faria - UERJ)

Jay Gopalakrishnan - Portland State University, USA
Título: Discontinuous functions in least-squares and DPG methods

Gustavo Buscaglia – USP, SP
Título: Finite elements for area-preserving fluid surfaces

Álvaro Coutinho, UFRJ, RJ
Título: Solving real life probabilistic edps under uncertainties: simulation of particle-laden turbulent flows

Philippe R. B. Devloo - FEC -Unicamp, SP
Título: On the development of high order Hdiv finite element approximation spaces including pyramid elements

Frederic Valentin - LNCC, RJ
Título: The mutiscale hybrid-mixed method: An overview and recent results

Segunda Sessão (Coordenadores: Maicon Correa e Sônia M. Gomes - Unicamp)

Mark Ainsworth - Brown University, USA
Título: Bernstein-Bezier Techniques in High Order Finite Element Analysis

Cristiane Faria – UERJ
Título: Stabilized Hybrid Descontinuous Galerkin Methods for Stokes and Elasticity Problems

Márcio Rentes Borges, LNCC, RJ
Título: Fluid flows in heterogeneous porous media: characterization of properties

Marcio Murad - LNCC, RJ
Título: A new iterative discrete-fracture-model for upscaling coupled flow and geomechanics in fractured reservoirs

Maicon Ribeiro Correa – IMECC
Título: New families of mixed finite elements on quadrilateral meshes

Discontinuous functions in least-squares and DPG methods

Jay Gopalakrishnan - Portland State University, USA

Abstract: A critical ingredient behind the design of methods like the Discontinuous Petrov Galerkin (DPG) methods is a finite element space of discontinuous functions. While such spaces are used as approximation spaces in standard methods like DG (discontinuous Galerkin) methods, the DPG methods use them for an essentially different purpose, namely stability. In DPG formulations, a finiteelement mesh is used not only to break the domain into elements, but also to break a Sobolev space. Standard Sobolev spaces has a broken analogue obtained by removing all interelement continuity constraints from its functions. When boundary value problems are reformulated using such broken spaces, interesting residual minimization processes become implementable. They are the basis for DPG methods. This talk will present DPG methods for Laplace and Maxwell equations, focusing on their least-squares minimization properties.

Finite elements for area-preserving fluid surfaces

Gustavo C. Buscaglia(1), Diego S. Rodrigues(2), Fernando Mut(3) and Roberto F. Ausas(1)

Abstract

At small length scales the physical response tends to be dominated by surface and interface behavior, which can be solid-like (e.g., elastic shells) or fluid-like (e.g. surfactant layers, lipid bilayers). The numerical modeling of biological processes at the Living Cell scale thus requires methods that correctly approximate the physics of the Cell's external membrane and numerous internal interfaces, which are modeled as area-preserving fluid surfaces.

This presentation discusses variational formulations for three-dimensional fluid flow and how they transform when a two-dimensional fluid sheet evolving in 3D space is considered. The area-preservation constraint, which is the surface analog to 3D incompressibility, is addressed by either Lagrange multipliers or by penalization, with linear triangles adopted for all unknown fields. There appear, as in most constrained problems, compatibility conditions between the intervening spaces. They are discussed in both the continuous and discrete settings, and a convergent scheme for equal-order elements is discussed.

Numerical results are focused on simulations of closed lipid-bilayers, which are capsules enclosed by a fluid surface that is endowed with bending energy. They show the good behavior of the proposed method in simulating relaxation and tweezing experiments.

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Solving real life probabilistic edps under uncertainties: simulation of particle-laden turbulent flows

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Abstract

Particle-laden flows are a very complex natural phenomenon. They are one of the processes for sediment transport and deposition that lead to the formation of basins hosting oil reservoirs. Those turbulent flows are triggered by small differences in the fluid density induced by the presence of sediment particles and usually modeled as a polydisperse mixture. A detailed modelling of this phenomenon may offer new insights to help geologists to understand the deposition mechanisms and the final stratigraphic form of the reservoirs. The increasing reliance on numerical simulation for the analysis of these complex physical systems has led in recent years to a strong development of knowledge in this area. In this sense, Uncertainty Quantification (UQ) provides a probabilistic framework to enable robust computer simulations that take into account the unavoidable uncertainties present in input parameters and in the model structure (model discrepancy). We will discuss UQ analysis, employing a probabilistic perspective, to consider the impact of using phenomenological models on quantities of interest such as bottom shear stresses and deposition maps. Those models combine experimental observations with physical intuition and try to capture the influence of the sediment concentration on the local flow viscosity and settling velocity. Both parameter uncertainties and different forms of model discrepancy (stochastic spatial fields) are considered in the simulations through non-intrusive stochastic collocation methods where several deterministic problems are solved to compute the required statistics. A scientific workflow management engine tool designed for high-performance computers supports the whole procedure.

On the development of high order Hdiv finite element approximation spaces including pyramid elements

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Abstract

This contribution relates the effort of the authors to develop H(div) conforming approximation spaces for a finite element with pyramidal topology. The development of these spaces for the pyramid element is particularly challenging because the H1 conforming spaces that form the basis of the H(div) spaces is composed of rational functions. As such the derivatives of the functions of order k are rational functions that are not necessarily included in the functions of order k-1.

Extensive use has been made of the software Mathematica to develop and verify the conformity of the spaces spanned by the rational polynomials formed by the divergence of the vector functions and the dual pressure functions. Thus far conforming polynomial spaces have been obtained for linear, quadratic and cubic fluxes.

The pyramid element is unique in that four faces converge to a point at its tip. Therefore, special care needs to be taken for the definition of the value of the flux at the tip of the pyramid. The proposed solution is to constrain the multiplying coefficient of one of the functions as a linear combination of the multiplying coefficients of the other faces. Convergence rates of finite element approximations composed of pyramidal and tetrahedral elements are presented to demonstrate the consistency of the approximation spaces.

Stabilized Hybrid Descontinuous Galerkin Methods for Stokes and Elasticity Problems

Cristiane O. Faria(1), Abimael F. D. Loula(2)

Abstract

Primal and mixed hybrid formulations are presented and analyzed for compressible, incompressible and near incompressible flow problems. The Stokes and the linear elasticity systems are considered as model problems. The proposed hybrid formulations preserve the main properties of the associate DG methods such as consistency, stability, boundedness and optimal rates of convergence in the energy norm, uniformly with respect to Poisson's ratio. Numerical results of convergence studies confirm the rates of convergence predicted by the numerical analysis.

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Bernstein-Bezier Techniques in High Order Finite Element Analysis

Mark Ainsworth - Brown University, USA

Abstract

We explore the use of Bernstein polynomials as a basis for finite element approximation on simplices in any spatial dimension. The Bernstein polynomials have a number of interesting properties that have led to their being the industry standard for visualisation and CAGD. It is shown that the basis enables the element matrices for the standard finite element space, consisting of continuous piecewise polynomials of degree n on simplicial elements in Rd, to be computed in optimal complexity O(n2d). The algorithms take into account numerical quadrature; are applicable to nonlinear problems; and do not rely on precomputed arrays containing values of one-dimensional basis functions at quadrature points (although these can be used if desired). The standard tools for the evaluation of Bezier curves and surfaces is the de Casteljau algorithm.

The archetypal pyramid algorithm is the de Casteljau algorithm. Pyramid algorithms replace an operation on a single high order polynomial by a recursive sequence of self-similar affine combinations and, as such, offer signficant advantages for high order finite element approximation. We develop and analyze pyramid algorithms for the efficient handling of all of the basic finite element building blocks, including the assembly of the element load vectors and element stiffness matrices. The complexity of the algorithm for generating the element stiffness matrix is optimal. A new, nonuniform order, variant of the de Casteljau algorithm is developed that is applicable to the variable polynomial order case but incurs no additional complexity compared with the original algorithm. The work provides the methodology that enables the efficient use of a completely general distribution of polynomial degrees without any restriction in changes between adjacent cells, in any number of spatial dimensions.

THE MULTISCALE HYBRID-MIXED METHOD: AN OVERVIEW AND RECENT RESULTS

FRÉDÉRIC VALENTIN

Abstract.
This work presents an overview of a family of finite element methods for multiscale problems, named Multiscale Hybrid-Mixed (MHM) methods. MHM methods are a consequence of a hybridization procedure which characterize the unknowns as a direct sum of a “coarse” solution and the solutions to problems with Neumann boundary conditions driven by the multipliers. As a result, the MHM method becomes a strategy that naturally incorporates multiple scales while providing solutions with high-order precision for the primal and dual variables. The completely independent local problems are embedded in the upscaling procedure, and computational approximations may be naturally obtained in a parallel computing environment. Also interesting is that the dual variable preserves the local conservation property using a simple post-processing of the primal variable.
Well-posedness and best approximation results for the one- and two-level versions of the MHM method show that the method achieves super-convergence with respect to the mesh parameter and is robust in terms of (small) physical parameters. Also, a face-based a posteriori estimator is shown to be locally efficient and reliable with respect to the natural norms. The MHM method, along with its associated a posteriori estimator, is naturally shaped to be used in parallel computing environments and appears to be a highly competitive option to handle realistic multiscale boundary value problems with precision on coarse meshes. The general framework and some recent results are illustrated for fluid flow models (Darcy and Stokes equations) and solid model (linear elasticity equation), a singularly perturbed transport problem (reactive-advective dominated equation), and a wave propagation problem (Maxwell equation). Particularly, we highlight how these MHM methods can be derived and analyzed within a common abstract setting, and we show a large varieties of numerical results for highly heterogeneous coefficient problems.

Keywords: finite element, hybridization, multiscale basis functions

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Fluid flows in heterogeneous porous media: characterization of properties

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Abstract

Natural porous media exhibit high degree of spatial variability in their hydraulic properties in multiple length scales. It has been established that such variability has a strong impact in determining fluid flow patterns in subsurface formations. Direct measurements of reservoir properties are only available at a small number of locations. The scarcity of available field data is partially due to the prohibitive cost of sampling processes associated with subsurface measurements. Once a deterministic description of the hydraulic parameters cannot be accomplished at all relevant locations, alternative conceptualizations suggest that a stochastic methodology is indispensable to treat rock properties. In practice, the characterization of such randomness is accomplished from statistics inferred by few experimental data (such as well data, log and core data or seismic data) and is commonly parametrized by two-point statistical moments such as mean and covariance. The underlying uncertainty in the description of reservoir parameters is a major factor that contributes to uncertainty in reservoir performance forecasting. In other words, the predictability of computational models is severely limited by the lack of an adequate description of formation properties. In recent years the increase in field data acquisition (although in limited points) associated with the use of high performing computing have encouraged the use of dynamic data, such as well test data, historical pressure data, fractional flow rate and measured concentration at the sensors, directly in the simulation of processes in order to reduce uncertainties and to improve the predictability of the models.

A NEW ITERATIVE DISCRETE-FRACTURE-MODEL FOR UPSCALING COUPLED FLOW AND GEOMECHANICS IN FRACTURED RESERVOIRS

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Abstract. A new computational model to describe flow and deformation in fractured reservoirs is rigorously constructed. The theory is developed within the framework of Biot's theory for poroelasticity for the matrix combined with a nonlinear elastic model for the network of discrete fractures which captures increase in stiffness during the fracture closure. The fully-coupled flow/geomechanics nonlinear formulation is constructed in the context of the Discrete Fracture Modeling and further rephrased within the framework of iteratively coupled methods where the hydrodynamics and mechanics subsystems are discretized in a sequential manner. Local profiles of velocity and pressure in the systems of matrix and fractures under the well known one-way formulation are numerically obtained. Considering realistic sections of the pre-salt reservoir in the Brazilian coast an straightforward upscaling methodology is developed to compute effective constitutive laws for properties such as permeability and porosity under oedometric conditions.

New families of mixed finite elements on quadrilateral meshes

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Abstract

In this work we present two new families of uniformly inf-sup stable mixed finite elements on quadrilateral meshes for approximating the flux and the scalar variables (**u**,p) of a second order elliptic equation in mixed form. Standard Raviart-Thomas (RT) and Brezzi-Douglas-Marini (BDM) elements are defined on rectangles and extended to quadrilaterals using the Piola transform, which are well-known to lose optimal approximation of the divergence of the flux. Arnold-Boffi-Falk (ABF) spaces rectify the problem by increasing the dimension of RT, so that approximation is maintained after Piola mapping. Our two families of finite elements are uniformly inf-sup stable, achieve optimal rates of convergence, and have minimal dimension. The elements for the flux are constructed from vector polynomials defined directly on the quadrilaterals, rather than being transformed from a reference rectangle by the Piola mapping, and then supplemented by two (one for the lowest order) basis functions that are Piola mapped. The two families are identical except for the inclusion of a minimal set of vector and scalar polynomials needed for higher order approximation of the divergence of the flux and the scalar variable. We present numerical results confirming the theory.