Numerical Challenges in PDE 2016
Part III

- **Date:** 13/09/2016
- **Venue:** Auditório do Laboratório de Computação Científica e Visualização Galileu, R. Josias Willard Gibbs s/n

Program

- **9:00-9:10 Opening section**
- **9:10-10:00 Plenary talk**
  - Prof. Mark Ainsworth, Brown University, USA
    High Order Finite Elements: Mathematitians's Playground or Practical Engineering Tool?
- **10:00-10:40 Poster section and coffee break**
  - Francisco T. Orlandini (Mestrado em Engenharia Elétrica, Unicamp) e P. R. B. Devloo (FEC, Unicamp)
    Development of HCurl approximation spaces and numerical modeling of wave guide problems
  - Manoucherh Sanei (Doutorado em Engenharia de Petróleo, Unicamp), P. R. B. Devloo (FEC, Unicamp) e Omar Durán Triana (Doutorado em Engenharia de Petróleo, Unicamp)
    A coupling between elasto-plastic damage analysis and permeability in porous media
  - Nathalia A. Batalha (Mestrado em Engenharia de Petróleo, Unicamp) e P. R. B. Devloo (FEC, Unicamp)
    On the numerical simulation of inclined wells
  - Roy Sanchez (Doutorado em Matemática– PUCP, Peru Lima)
    Approximation of singular problems associated with the Navier Stokes equations
- **10:40-12:10 Oral presentations**
  - Maicon R. Correa (IMECC, Unicamp)
    Mixed Finite Elements for Porous Media Flow
  - Philippe R. B. Devloo (FEC, Unicamp)
    The Multiscale Hybrid Method in the context of mixed finite element approximations
  - Giuseppe Romanazzi (IMECC, Unicamp)
    Numerical Simulation and Modelling of the firsts stages of Colorectal Cancer
- **12:10-14:00 Lunch break**

- **14:00-15:30 Oral presentations**
  - Omar Durán Triana (Doutorado em Engenharia de Petróleo, Unicamp) e P. R. B. Devloo (FEC, Unicamp)
    Development of a Multi-scale Three-Phase Reservoir Simulator Coupled with Geomechanics
  - Sônia M. Gomes (IMECC, Unicamp)
    H(div) hp-adaptive finite element approximations on curved geometries
  - Thiago D. Santos (Doutorado em Engenharia Civil, Unicamp), Mathieu Morlighem (UCI, USA), Hélène Serouss, Eric Larour (JPL/Caltech, USA), P. R. B. Devloo (FEC, Unicamp) e J. C. Simões (UFRGS)
    h-Adaptivity Applied to Ice Sheet Simulation
- **15:30-16:10** Poster section and coffee break

Abstracts of the contributions

HIGH ORDER FINITE ELEMENTS: MATHEMATICIAN'S PLAYGROUND OR PRACTICAL ENGINEERING TOOL?

MARK AINSWORTH

Division of Applied Mathematics, Brown University, Providence RI 02912
E-mail address: MarkAinsworth@brown.edu

Abstract

High order finite element methods have been analysed ex- tensively for a wide variety of applications and are known to be capa- ble of producing exponential rates of convergence, even for challenging problems with singularities, sharp boundary layers and high frequency oscillations. High order polynomial approximations are commonplace in many areas of scientific computing including computer graphics, com- puter aided-geometric design, and spectral methods for PDEs. It is commonplace to see the spectral method used with approxima- tion orders in the 100s or even 1000s. Yet, despite theory giving the nod to the use of very high order finite element methods, the range of polynomial degree used in practical finite element computations is rarely larger than eighth order! Few commercial codes allow the use of high order finite elements. The rather modest polynomial degrees seen in high order finite element analysis are due to efficiency considerations rather than any theoretical barriers. Bernstein-B´ezier polynomials have a number of interesting properties that have led to their being the industry standard for visualisation and CAGD. We explore the use of Bernstein polynomials as a basis for finite element approximation.

Mixed Finite Elements for Porous Media Flow

Maicon R. Correa

IMECC, Unicamp

In this talk we discuss the application of different families of uniformly inf-sup stable mixed finite elements on quadrilateral meshes for approximating the velocity and the pressure (u,p) of a second order elliptic equation in mixed form, with particular interest in porous media flow.

The Multiscale Hybrid Method in the context of mixed finite element approximations

P. R. B. Devloo – FEC, Unicamp

In this talk we present the Multiscale Hybrid Method (MHM) as idealized by F. Valentin and D. Paredes [1], and reinterpret the method in the case where the numerical approximation in the macro domains is performed with mixed finite elements.
The MHM approximations with mixed finite elements amount to applying shape function restraints to the original fine mesh approximation. The systematic of shape function restraints is the same as the one that is used when using hp-adapted meshes.
The resulting approximation technique is local conservative. The equations of the subdomains can be condensed on the equations of the interface fluxes.
The structure of the MHM-M method is presented in the context of the NeoPZ finite element programming environment.

[1] F. Valentin C. Harder, D. Paredes. A family of multiscale hybrid-mixed finite element methods for the darcy equation with rough coefficients. J. Comput. Phys., 245:107–130, 2013.

Numerical Simulation and Modelling of the firsts stages of Colorectal Cancer

Giuseppe Romanazzi

IMECC, Unicamp

Abstract

Colorectal cancer is one of the most frequent type of cancer in the Western World. Different scientific communities have studied its morphogenesis from geneticists to computing scientists. The colon is a suitable place for the appearance of cancer because of its continuous self-renewal with a large number of cell divisions per day. The inner part of the colon is lined by millions of small pits, called crypts and it is widely accepted that the cell mutations in the crypts are responsible for the cancer initiation process. It is consensual in the medical community, that a potential first manifestation of the carcinogenic process, observed in conventional colonoscopy images, is the appearance of Aberrant Crypt Foci (ACF). These are clusters of abnormal crypts, morphologically characterized by an atypical behavior of the cells that populate the crypts. In this presentation, I describe a periodic multiscale and a homogenization model for the ACF dynamics based on the cellular dynamics in the colon epithelium. The goal is to simulate and predict the spread and evolution of ACF, as it can be observed in colonoscopy images. The methods implemented to solve the multiscale and the homogenization models belong to the Heterogeneous Multiscale Methods (HMM) framework and it is implemented using a Finite Elements discretization. Numerical results showing the convergence of the periodic model to the homogenized model are presented.

Development of a Multi-scale Three-Phase Reservoir Simulator Coupled with Geomechanics

Omar Duran Triana[a], Philippe Remy Bernard Devloo[b]

[a]*Department of Mechanical Engineering, State University of Campinas, Campinas*
[b]*Department of Civil Engineering, State University of Campinas, Campinas*

Abstract

The scope of this work is to development a efficient finite element finite volume scheme, that can solve fine detailed geological realizations by the use of a new multiscale approach coupled with geomechanics on structured and non structured grids, it combined with reduced order modelling (ROM) methods allows the fast generation of surrogate reservoir models. The strong formulation being considered is the weighted pressure formulation for the black-oil coupled with Biot poro-elasticity model. Using a finite element - finite volume discretization scheme that stores/transfers all the required data at integration points and the multi-scale hybrid mixed method MHM, it is possible to solve efficiently conservative fluxes on both coarsened and fine scales. Once theses fluxes are determined, they are used to solve transport of the flowing phases. The nonlinearities associated with black-oil model are treated with a segregated parabolic-hyperbolic multi-physics solver without mass discrepancy. We present evaluations for computational efficiency and accuracy on a series of full and corresponding reduced order reservoir models with a high degree of realism, highly heterogeneous rock properties distributed in an representative domain coupled with geo-mechanical reservoir deformation. The multi-scale finite element method simulation with integration point transfers system, allow us to quickly compute mono-phasic simulation states that feeds ROM procedures in the geo-mechanic coupling and it represents a new and robust approach that enable us to generate reduced order spaces to simulate a detailed geological realizations not allowed be possible with current up-scaling methods.

Keywords: Reservoir Simulation, Black-Oil model, Mixed Formulations, Segregated Solver

[1] Christopher Harder, Diego Paredes, Frdric Valentin, A family of Multiscale Hybrid-Mixed finite element methods for the Darcy equation with rough coefficients, Journal of Computational Physics, Volume 245, 15 July 2013, Pages 107-130, ISSN 0021-9991, http://dx.doi.org/10.1016/j.jcp.2013.03.019.

[2] John A. Trangenstein and John B. Bell. Mathematical Structure of the Black-Oil Model for Petroleum Reservoir Simulation, SIAM Journal on Applied Mathematics.

H(div) hp-adaptive finite element approximations on curved geometries

Sônia M. Gomes - IMECC, Unicamp

The mixed finite element formulation for elliptic problems is characterized by simultaneous calculations of the potential (primal variable) and of the flux field (dual variable). This work focuses on new H(div)-conforming finite element spaces, which are suitable for flux approximations. Following the developments in [1], for 3D affine elements, and in [2] for 2D curved elements, we shall consider curved meshes in 3D using tetrahedral, hexahedral or prismatic elements. The adopted methodology for the construction of H(div) bases consists in using hierarchical H^1 conforming scalar bases multiplied by vector fields that are properly constructed on the master element and mapped to the geometric elements by the Piola transformation. Two stable approximation space configurations are treated. Their choices are guided by the property that, in the master element, the image of the flux space by the divergence operator coincides with the primal space. Numerical results are presented for a test problem with smooth solutions using 3D regular curved meshes showing order $k+1$ or $k+2$ for the primal variable, while keeping order $k+1$ for the flux in both configurations. H(div)-conforming finite element spaces based on curved quadrilateral meshes, with $hp-$adaptation are also applied in the simulation of a test problem with square-root singularity at a boundary point. The results demonstrate exponential rates of convergence, and a dramatic error reduction when quarter-point elements are applied close to the singularity. Using static condensation, we show that the global condensed matrices to be solved have reduced dimension, which is proportional to the dimension of border fluxes.

References

[1] D. A. Castro, P. R. B. Devloo, A. M. Farias, S. M. Gomes, D. de Siqueira, O. Durán. Three dimensional hierarchical mixed finite element approximations with enhanced primal variable accuracy. Comput. Meth. Appl. Mech. and Eng. 306 (2016) 479–502.

[2] Castro, D. A., Devloo, P. R. B., Farias, A. M., Gomes, S. M., & Durán, O., 2016b. Hierarchical high order finite element bases for H(div) spaces based on curved meshes for two-dimensional regions or manifolds. Journal of Computational and Applied Mathematics, vol. 301, pp. 241– 258.

h-Adaptivity Applied to Ice Sheet Simulation

Thiago Dias dos Santos
Mathieu Morlighem
Hélène Seroussi
Eric Larour
Jefferson Cardia Simões
Philippe Remy Bernard Devloo

This work proposes to make contributions in ice sheet modeling. Ice sheets have received attention by national and international scientific community in recent decades because of its potential contribution to sea level rise. This work aims to contribute in ice sheet modeling by applying *h*-adaptive refinement and goal-oriented error estimates with the ISSM software (Ice Sheet System Model). ISSM is developed by Jet Propulsion Laboratory at the California Institute of Technology (JPL/Caltech) and by the University of California at Irvine (UCI). The algorithm of *h*-adaptive refinement is based on the finite element library NeoPZ, developed by the Computational Mechanics Laboratory at UNICAMP. The set of partial differential equations of ice sheet evolution and the numerical scheme to solve them are presented. Numerical experiments based on the Marine Ice-Sheet Model Intercomparison Project (MISOMIP) are simulated using ISSM and NeoPZ. Results using *h*-adaptive refinement show reduction of the numerical error and computational cost.

Implementation of an H_{curl} conforming approximation space on the *NeoPZ* environment

Francisco Teixeira Orlandini — State University of Campinas

francisco.orlandini@gmail.com

The H_{curl} approximation space has been widely used on the Computational Electromagnetics area, for the need of the magnetic and electric fields to have tangential continuity between different media. The poster will describe the efforts to implement such an approximation space in the *NeoPZ* environment from existent H_{div} functions in order to work on waveguide modal analysis. Challenges regarding the Nédélec restrictions, among others, will be dwelled upon, and the next steps will be detailed.

A coupled between elasto-plastic damage analysis and permeability in porous media

Manouchehr Sanei[a] , Philippe R.B. Devloo[b] and Omar Dur´an Triana[c]

Abstract:

Coupled flow and deformation analysis in porous media is a significant subject in some specific engineering applications, including: reservoir engineering, enhancement oil recovery, petroleum drilling, nuclear waste storage, geothermal energy, etc. As far as is concerned, the purpose of this paper is to develop a simplified numerical approach for coupling elasto-plastic damage and permeability in porous media. The deformation model is established using the Mohr-Coulomb model, corresponding the appropriate backward Euler-based return mapping, and the conservation of mass and momentum by considering the Darcy's fluid flow law. In this study the numerical approach is developed based on the Galerkin finite element method (GFEM). The proposed methodology is verified and validated by comparison with experimental data. The results show that the outcome method is suitable for coupled analysis of flow and deformation in porous media.

Keywords Porous media; Elasto-plastic analysis; Permeability; Mohr Coulomb model; Coupled flow deformation; Finite Element Method

[a] DEP - FEM - Unicamp
E-mail: sanei@dep.fem.unicamp.br

[b] FEC - Unicamp
E-mail: phil@fec.unicamp.br

[c] DEP - FEM – Unicamp
E-mail: omar@dep.fem.unicamp.br

On the numerical simulation of inclined wellbores

by

Nathalia Agostinho Batalha
Philippe Remy Bernard Devloo
LABMEC - UNICAMP

Abstract:
One of the greatest concerns in drilling operations is wellbore stability. Stuck pipe, hole collapse, cavity enlargement, formation fracturing, time loss and substantial over cost are some of the consequences caused by unstable wells. Drilling through a formation affects the state of stress of the previous medium. Although the mud drilling temporarily supports the borehole wall during the operation, an unavoidable new stress distribution occurs due the relief induced by the cavity. This new state of stress may lead to rock failure. The formation response after drilling is even less favorable when dealing with inclined wellbores. Thus, a two-dimension numerical modeling was developed to predict stress state around inclined wells. The present model assumes linear elastic and isotropic rock material behavior with plane strain state. This geomechanical model was elaborated using the finite elements method. The method was implemented with NeoPZ, which is an open-source library for development of finite element simulations. The simulation provides the user a view of the simulation result over the plane perpendicular to the inclined hole or projected over a horizontal cross section. For result accuracy, analytic stress solution may also be used to define initial stress through the domain and as boundary conditions. The numerical solution demonstrated good outcome when compared to analytical equations indicating a suitable approach.

Keywords: inclined wellbores, state of stress, numerical modeling, finite element method.

Turbulent Flow and Niever-Stokes Equations

Roy W. Sánchez G.*
PUCP (Perú)

Abstract

Introduction to turbulence from laminar flow, studying the stability criteria, the rotational flow in cylindrical coordinates and the turbulence model for Reynolds stresses. The turbulence is initiated by flow instability, originated (this instability) by gradients average speeds. Moving vortices generate new instabilities leading to smaller eddies. This process continues until disappear by the loss of energy through the viscosity. We try to explain the phenomenon of turbulence as a particular case of the Navier-Stokes equations with the concepts of instability. Since there is no mathematical model that provides adequate turbulence problems, mainly due to fluctuations over time that generates a lot of uncertainty and chaos solution.

Keywords

Vortex flow, stability, number of Reynolds, Navier-Stokes equations and turbulence.

References

[1] AKIRA OGAWA: *Vortex Flow.* CRC Press; 2000. Department of Mathematics University Louisiana State University.

[2] CHORIN, W.-MARSDEN, J.: *A Mathematical Introduction to fluid Mechanics.* Springer-Verlag; 1992. Department of Mathematics University of Texas at Austin.

[3] DONEA, J.- HUERTA, A.: *Finite Element Methods for Fluw Problems.* John Wiley; 2003.

[4] ELMAN, H.: *Finite Elements and Fast Iterative.* Oxford; 2006. Department of Mathematics Oxford University Press.

[5] EVANS, l.: *Partial Differential Equations.* American Mathematical Society; 2000. Department of Mathematics University of California, Berkeley.

*rwsanche@pucp.edu.pe