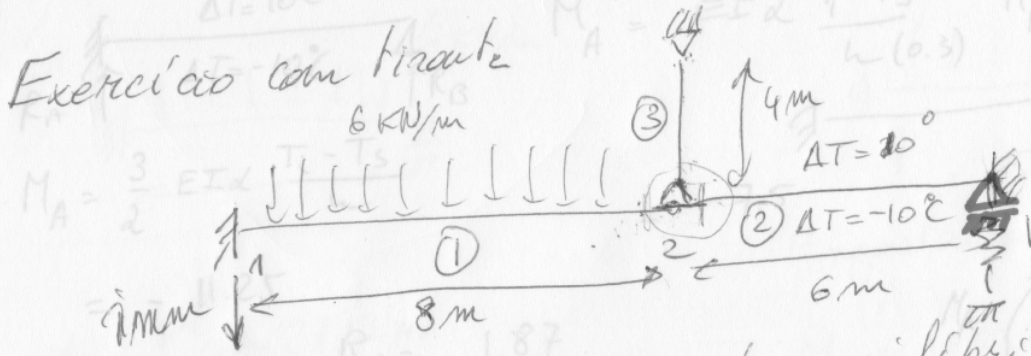
Focalizar

Exercício com tirante



① $E_{Aço} = 209\,000$ MPa

$E_{conc} = 25\,000$ MPa

$A_{tirante} = \frac{1}{2}$ cm²

Viga 30 cm × 20 cm

$\alpha = 10^{-5}$ /°C

$K_{mola} = 5$ KN/cm

$I = \frac{0.2\,(0.3)^3}{12}$ [m]⁴ $= 4.5\,10^{-4}$ [m]⁴

$EI = 25\,10^6\, I$ [KN m²]

$= 11\,250$ KN/m²

Graus de liberdade

$V_2\ (D_1) \rightarrow$

$\varphi_2\ (P_2) \rightarrow$

$V_3\ (D_3) \rightarrow$

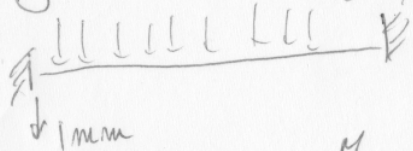
Equação de equilíbrio

$R_B^{(1)} + R_B^{(3)} + R_A^{(2)} = 0$

$M_B^{(1)} + M_A^{(2)} = 0$

$R_B^{(2)} + K V_3 = 0$

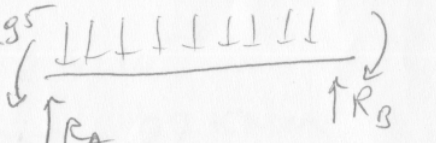
① Carregamentos externos



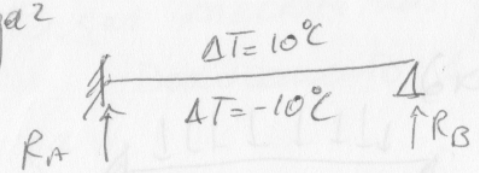
Carga distribuída: $M_A = 32$ $M_B = -32$

recalque $M_A = \frac{-6EI}{l^2}\Delta = -1.05$ $M_B = \frac{-6EI}{l^2}\Delta = -1.05$

Total $M_A = 30.95$ $M_B = -33.05$



Viga 2



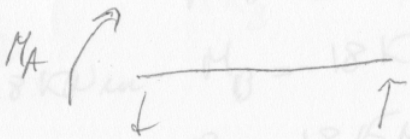
$$M_A = \frac{3}{2} EI\alpha \frac{T_i - T_s}{h}$$

$$= -11.25$$

$$R_A = -1.87 \quad R_B = 1.87$$

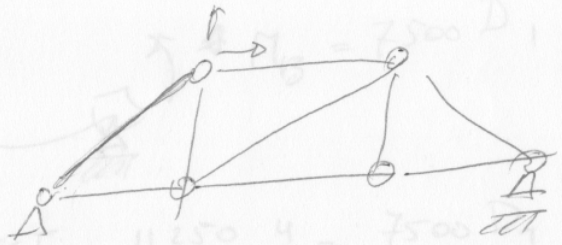
$$M_A^{EP} = EI\alpha \frac{T_i - T_s}{h(0.3)} \quad M_B^{EP} = -EI\alpha \frac{T_i - T_s}{h}$$

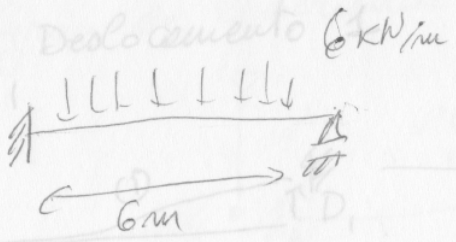
$$= -7.5 \qquad = 7.5$$



tirante

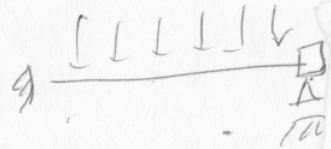
$$R_A = H_A = R_B = H_B = 0$$





$$M_A = \frac{PL^2}{8} = 27\ KN m$$

$$R_A = \cancel{7.5}\ 22.5 \qquad R_B = 13.5\ KN$$



Engastamento perfeito

$$M_A = 18\ KN m \qquad M_B = -18\ KN m$$

$$R_A = 18\ KN \qquad R_B = 18\ KN$$

$$M_B = \frac{4EI}{L} = \frac{11250\ 4}{6} = 7500\ D_1$$

$$M_A = \frac{2EI}{L} = 3750\ D_1$$

Equação de equilíbrio

$$M_B + 7500\ D_1 = 0$$

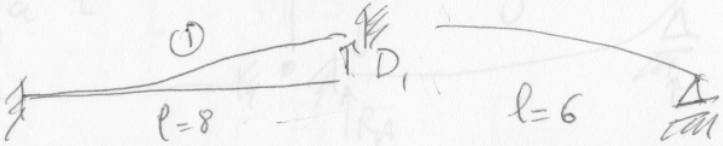
$$D_1 = \frac{18}{7500} = 2.4\ 10^{-3}$$

$$M_A^{Total} = 18 + 2.4\ 10^{-3} \cdot 3750 = 27\ KN m$$

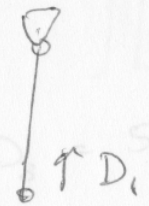
Forças necessários para sustentar os deslocamentos

Deslocamento 1

Viga 1



Viga 2



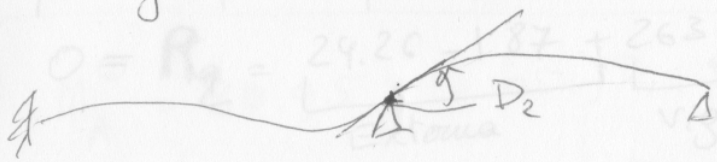
$\uparrow R_3 = D_1 \frac{EA}{2} = 2562\, D_1$

$R_B^{①} = \frac{12EI}{\ell^3} = 263.\, D_1$ $\quad R_A^{②} = \frac{3EI}{\ell^3} = 156\, D_1$

$M_B^{①} = \frac{-6EI}{\ell^2} = -1054\, D_1$ $\quad M_A^{②} = \frac{3EI}{\ell^2} = 937\, D_1$

$R_B^{②} = -156\, D_1$

Giro em 2 : Grau de liberdade 2

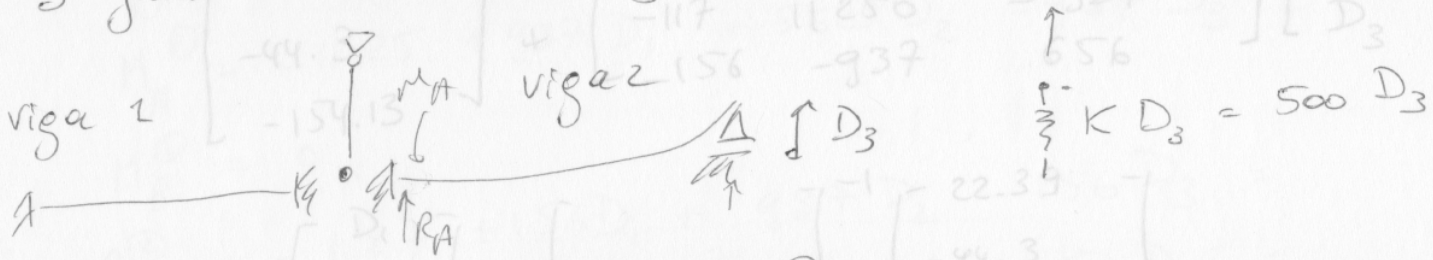


$M_B^{①} = \frac{4EI}{L} D_2 = 5625\, D_2$ $\quad M_A^{②} = \frac{3EI}{L} D_2 = 5625\, D_2$

$R_B^{①} = \frac{-6EI}{L^2} D_2 = -1054\, D_2$ $\quad R_A^{②} = \frac{3EI}{L^2} = 937\, D_2$

$R_B^{②} = -\frac{3EI}{L^2} = -937\, D_2$

3° Grau de liberdade V_3 (D_3)



$$R_A = -\frac{3EI}{\ell^3} D_3 = -156 D_3$$

$$R_B = \frac{3EI}{\ell^3} D_3 = 156 D_3$$

$$M_A = -\frac{3EI}{\ell^2} D_3 = -937 D_3$$

$-156 D_3$

Equações de equilíbrio

$$0 = R_2 = \underbrace{29.26 - 1.87}_{\text{Externa}} + \underbrace{263 D_1}_{\text{viga 1}} + \underbrace{156 D_1}_{\text{viga 2}} + \underbrace{2562 D_1}_{\text{Finaute}} + \underbrace{(-1054) D_2}_{\text{Viga 1}} + \underbrace{937 D_2}_{\text{viga 2}}$$

$$0 = M_2 = \underbrace{-33.05}_{\text{viga 1}} \underbrace{-11.25}_{\text{viga 2}} + \underbrace{(-1054) D_1}_{\text{viga 1}} + \underbrace{937 D_1}_{\text{viga 2}} + \underbrace{5625 D_2}_{\text{viga 1}} + \underbrace{5625 D_2}_{\text{viga 2}} - 937 D_3$$

$$0 = R_3 = \underbrace{1.87}_{\text{viga 2}} \underbrace{-156 D_1}_{\text{viga 2}} \underbrace{-937 D_2}_{\text{viga 2}} + \underbrace{156 D_3}_{\text{viga 2}} + \underbrace{500 D_3}_{\text{mola}}$$

$$\begin{bmatrix} 22.39 \\ -44.3 \\ -154.13 \end{bmatrix} + \begin{bmatrix} 2981 & -117 & -156 \\ -117 & 11250 & -937 \\ -156 & -937 & 656 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = -\begin{bmatrix} \quad \end{bmatrix}^{-1} \begin{bmatrix} 22.39 \\ -44.3 \\ -154.13 \\ -1.87 \end{bmatrix}$$

~~$D_1 = -0.00693$~~

$D_1 = -0.0073$

$D_2 = 0.00395$

$D_3 = 0.00105$

$$M_A^{①} = 30.95 + (-1054) D_1 + \frac{5625}{2} D_2 =$$

$$M_B^{①} = -33.05 + (-1054) D_1 + \frac{5625}{2} D_2 =$$

$$R_A^{①} = 23.74 - 263 D_1 + 1054 D_2 =$$

$$R_B^{①} = 24.26 - 263 D_1 - 1054 D_2 =$$

Viga 2:

$$M_A^{②} = -11.25 + 937 D_1 + 5625 D_2 - 937 D_3 =$$

$$M_B^{②} = 0$$

$$R_A^{②} = -1.87 + 156 D_1 + 937 D_2 - 156 D_3 =$$

$$R_B^{②} = 1.87 - 156 D_1 - 937 D_2 + 156 D_3 =$$

Tirante

$$N = \cancel{-500} + 2562 D_1$$

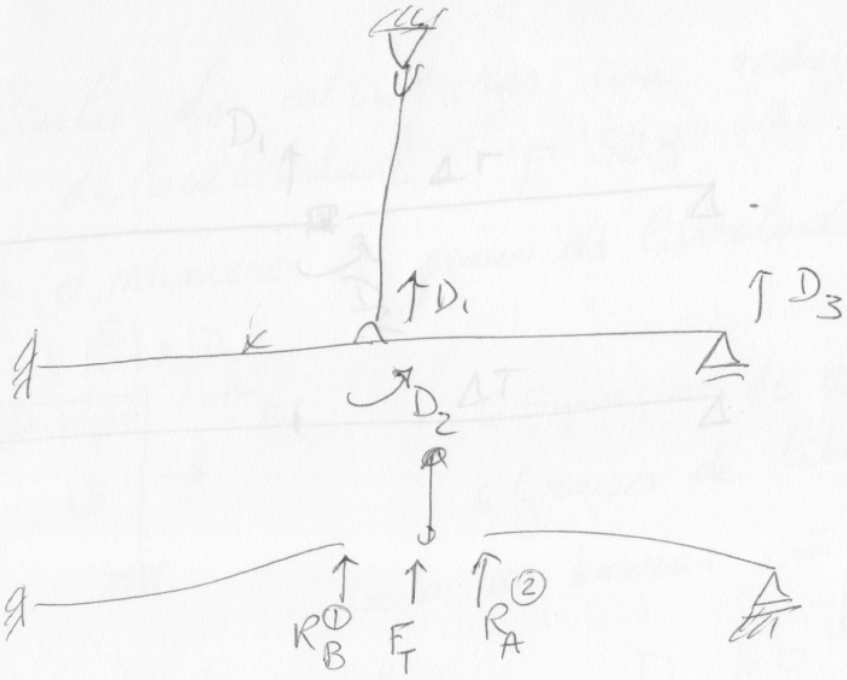
Força na mola

$$F = 500 D_3$$

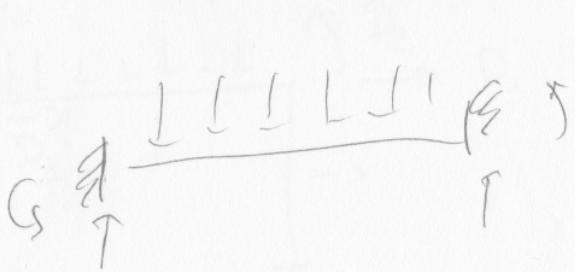
Verificação: $M_{.B}^{①} + M_A^{②} = 0$

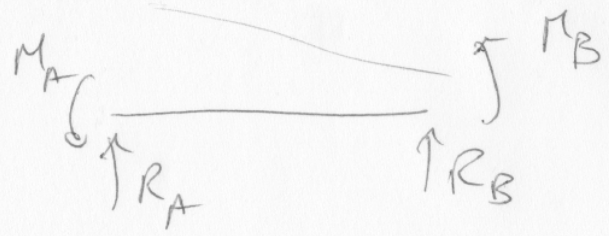
$$R_B^{①} + R_A^{②} + (-N) = 0$$

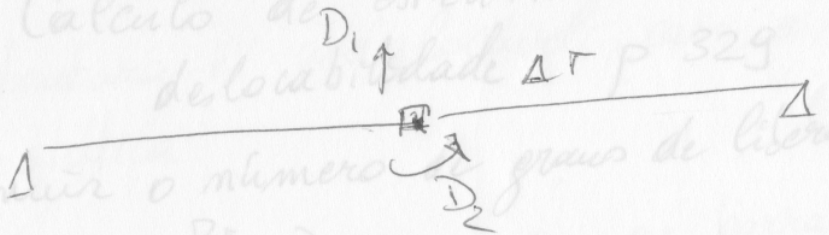
$$R_B^{②} + K D_3 = 0$$

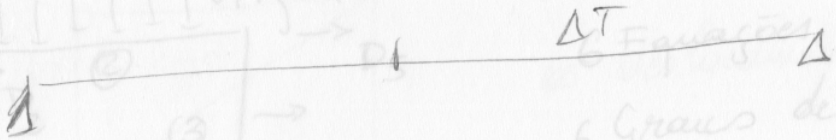


$$R_2 = R_B^{(1)} + R_A^{(2)} + F_T = 0$$






D1
ΔT
D2


ΔT