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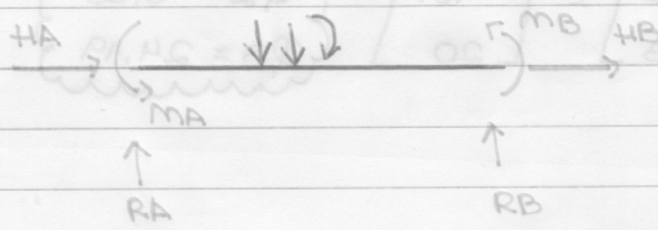
Método dos deslocamentos



$$M_A = M_{AB} + \beta_{AB}\varphi_A + \alpha_{BA}.\beta_{BA}.\varphi_B + \beta_{AB}\frac{(1+\alpha_{AB})}{L}(v_A - v_B)$$

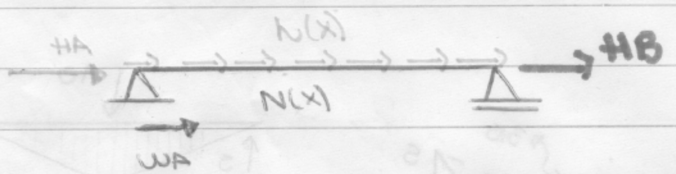
$$M_B = M_{BA} + \beta_{AB}.\alpha_{AB}.\varphi_A + \beta_{BA}\varphi_B + \beta_{BA}\frac{(1+\alpha_{BA})}{L}(v_A - v_B)$$

$$R_A = R_{AB} + \frac{(M_A - M_{AB} + M_B - M_{BA})}{L}$$

$$R_B = R_{BA} - \frac{(M_A - M_{AB} + M_B - M_{BA})}{L}$$

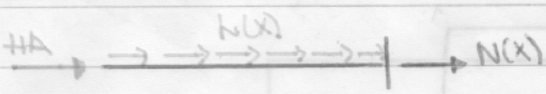
$$H_A = f_A(H_{AB}, w_A, w_B)$$

$$H_B = f_B(H_{BA}, w_A, w_B)$$



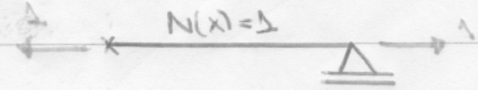
$$H_A = -\int_0^L h(x).dx - H_B$$

$$N(x) = -H_A - \int_0^x h(t)\,dt$$



- Cálculo de w_B

PTV: Estado de Carregamento:



spiral

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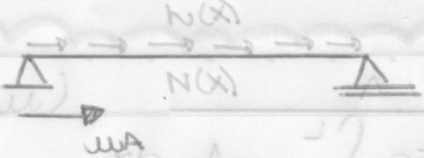
Estado de deformação:



$TV^{ext} = \bar{u}_B - \bar{u}_A$

$TV^{int}: \int_0^L 1 \times \frac{N(x)}{EA(x)} \, dx$

$N(x) = -H_A - \int_0^x h(t)\,dt = H_B + \int_0^L h(x)\,dx - \int_0^x h(t)\,dt$

$H_T \rightarrow$ integral de todas as forças internas atuando sobre a viga

$\bar{u}_B - \bar{u}_A = \int_0^L \frac{N(x)}{EA} \, dx$

$u_B = u_A + E_cI_c \int_0^L \frac{N(x)}{EA} \, dx$

$u_B = u_A + E_cI_c \int_0^L \frac{\left(H_B + H_T - \int_0^x h(t)\,dt\right)}{EA} \, dx$

$$\boxed{u_B = u_A + E_cI_c \, (H_B + H_T) \int_0^L \frac{1}{EA} \, dx - E_cI_c \int_0^L \frac{\int_0^x h(t)\,dt}{EA} \, dx}$$

$$\bullet \; H_B = H_{BA} + \frac{1}{E_cI_c \int_0^L \frac{1}{EA} \, dx} (u_B - u_A)$$ "global"

para vigas prismáticas:

$$H_B = H_{BA} + \frac{EA}{L \cdot I_cI_c} (u_B - u_A)$$

$* \; H_A + H_B + H_T = 0$

$H_A = -H_B - H_T$

$$H_A = \underbrace{\left(-H_{BA} - H_T\right)}_{\text{Chamar de } H_{AB}} - \frac{1}{E_cI_c \int_0^L \frac{1}{EA} \, dx} (u_B - u_A)$$

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$$H_A = H_{AB} + \frac{1}{E_cI_c \int_0^L \frac{1}{EA}\,dx}\,(u_A - u_B)$$ "GLOBAL"

$$H_A = H_{AB} + \frac{EA}{L\,E_cI_c}\,(u_A - u_B)$$ para vigas prismáticas

Resumindo:

$$H_A = H_{AB} + \frac{1}{E_cI_c \int \frac{1}{EA}}\,(u_A - u_B)$$

$$H_B = H_{BA} + \frac{1}{E_cI_c \int \frac{1}{EA}}\,(u_B - u_A)$$

$$\begin{bmatrix} H_A \\ R_A \\ M_A \\ H_B \\ R_B \\ M_B \end{bmatrix} = \begin{bmatrix} H_{AB} \\ R_{AB} \\ M_{AB} \\ H_{BA} \\ R_{BA} \\ M_{BA} \end{bmatrix} + \begin{bmatrix} \frac{1}{E_cI_c\int\frac{1}{EA}} & 0 & 0 & \frac{-1}{E_cI_c\int\frac{1}{EA}} & 0 & 0 \\ 0 & K_{22} & \frac{\beta_{AB}(1+\alpha_{AB})}{L} & 0 & -K_{22} & \frac{\beta_{BA}(1+\alpha_{BA})}{L} \\ 0 & \frac{\beta_{AB}(1+\alpha_{AB})}{L} & \beta_{AB} & 0 & -\frac{\beta_{AB}(1+\alpha_{AB})}{L} & \beta_{BA}\alpha_{BA} \\ \frac{-1}{E_cI_c\int\frac{1}{EA}} & 0 & 0 & \frac{1}{E_cI_c\int\frac{1}{EA}} & 0 & 0 \\ 0 & -K_{22} & -\frac{\beta_{AB}(1+\alpha_{AB})}{L} & 0 & K_{22} & \frac{\beta_{BA}(1+\alpha_{BA})}{L} \\ 0 & \frac{\beta_{BA}(1+\alpha_{BA})}{L} & \beta_{BA}\alpha_{BA} & 0 & -\frac{\beta_{BA}(1+\alpha_{BA})}{L} & \beta_{BA} \end{bmatrix} \begin{bmatrix} u_A \\ v_A \\ \varphi_A \\ u_B \\ v_B \\ \varphi_B \end{bmatrix}$$

$$* \; K_{22} = \frac{\beta_{AB}(1+\alpha_{AB}) + \beta_{BA}(1+\alpha_{BA})}{L^2}$$

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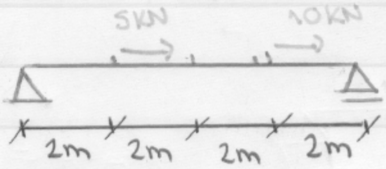
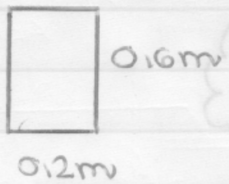
para vigas prismáticas: matriz simétrica [K]

$$
\begin{bmatrix} HA \\ RA \\ MA \\ HB \\ RB \\ MB \end{bmatrix} =
\begin{bmatrix} HAB \\ RAB \\ MAB \\ HBA \\ RBA \\ MBA \end{bmatrix}
\begin{bmatrix}
\frac{EA}{LEcIc} & 0 & 0 & \frac{-EA}{LEcIc} & 0 & 0 \\
0 & \frac{12}{L^2L'} & \frac{6}{LL'} & 0 & \frac{-12}{L^2L'} & \frac{6}{LL'} \\
0 & \frac{6}{LL'} & \frac{4}{L'} & 0 & \frac{-6}{LL'} & \frac{2}{L'} \\
\frac{-EA}{LEcIc} & 0 & 0 & \frac{EA}{LEcIc} & 0 & 0 \\
0 & \frac{-12}{L^2L'} & \frac{-6}{LL'} & 0 & \frac{12}{L^2L'} & \frac{-6}{LL'} \\
0 & \frac{6}{LL'} & \frac{2}{L'} & 0 & \frac{-6}{LL'} & \frac{4}{L'}
\end{bmatrix}
\begin{bmatrix} uA \\ vA \\ \varphi A \\ uB \\ vB \\ \varphi B \end{bmatrix}
$$

Exercício: Cálculo da rigidez axial e de HAB, HBA





$E = 25000$ MPa

$EcIc = 10^5$ kN.m²

① caracterização da viga

*Rigidez Axial $\dfrac{1}{EcIc\int\frac{1}{EA}} = \dfrac{EA}{L\,EcIc} = \dfrac{30}{8} = 3{,}75$

$EA = 25 \cdot 10^6 \cdot 0{,}2 \cdot 0{,}15 = 3 \cdot 10^6$

$EI = 25 \cdot 10^6 \cdot \dfrac{0{,}2 \cdot 0{,}15^3}{12} = 90 \cdot 10^3$

$L' = 8$ $\quad$ $*\,L'\dfrac{EcIc}{EI} = \dfrac{8 \cdot 10^5}{9 \cdot 10^4} = 8{,}8$ $\quad$ $*\,\beta = \dfrac{4}{8{,}8} = 0{,}45$

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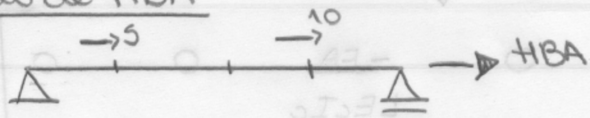
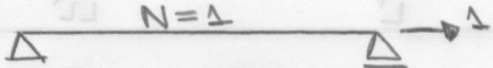
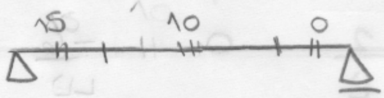
* Cálculo de HBA:



↳ Estado de carregamento:



↳ Estado de Deformação:



(se não existisse HBA)

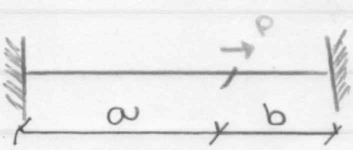
$$\delta_{10} = \int_0^8 \frac{du}{dx} \cdot N \, dx = \int_0^2 \frac{15}{EA} \, dx + \int_2^6 \frac{10}{EA} \cdot dx = \frac{70}{EA}$$

$$\delta_{11} = \int_0^8 \frac{1}{EA} \, dx = \frac{8}{EA}$$

$$\delta_1 = \delta_{10} + HBA \cdot \delta_{11} = 0 \Rightarrow HBA = -\frac{70}{8} = -8{,}75 \quad HBA = -8{,}75$$

$$HAB + HBA + 15 = 0 \quad HAB = -6{,}25$$

• Intuitivamente



$$HAB = -\frac{P.b}{L} \quad HBA = -\frac{P.a}{L}$$

Conferindo o exercício

$$HAB = -\frac{5.6}{8} - \frac{10.2}{8} = -\frac{50}{8} = -6{,}25$$

$$HBA = -\frac{5.2}{8} - \frac{10.6}{8} = -\frac{70}{8} = -8{,}75$$

OK!

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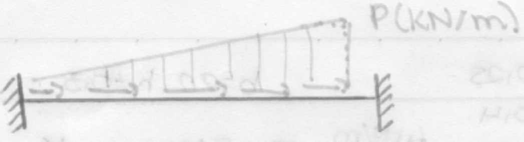
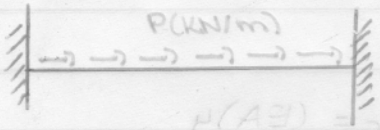


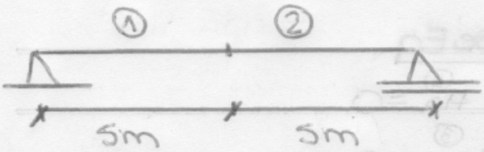
$$\boxed{H_{AB} = -\frac{PL}{12} \qquad H_{BA} = -\frac{PL}{6}}$$

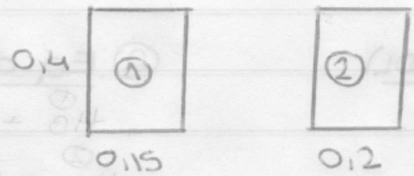


$$\boxed{H_{AB} = -\frac{PL}{2} \qquad H_{BA} = -\frac{PL}{2}}$$

Exercício Calcular rigidez axial

$E = 25 \cdot 10^6 \, KN/m^2$





$E_cI_c = 10^5 \, KNm^2$

$A_1 = 0{,}06 \qquad EA_1 = 1{,}5 \cdot 10^6 \, KN$

$A_2 = 0{,}12 \qquad EA_2 = 3 \cdot 10^6 \, KN$

* Rigidez Axial: $\dfrac{1}{E_cI_c \int \frac{1}{EA} dx} = \dfrac{1}{E_cI_c\left(\frac{5}{EA_1} + \frac{5}{EA_2}\right)}$

$$= \frac{1}{5\left(\frac{1}{15} + \frac{1}{30}\right)} = \frac{1}{\frac{5}{30} \cdot 3} = 2$$