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$$M_A = M_{AB} + \beta_{AB}\cdot\varphi_A + \beta_{BA}\,\alpha_{BA}\,\varphi_B + \beta_{AB}\left(\frac{1+\alpha_{AB}}{L}\right)(V_A - V_B)$$

$$M_B = M_{BA} + \beta_{AB}\cdot\alpha_{AB}\cdot\varphi_A + \beta_{BA}\,\varphi_B + \beta_{BA}\left(\frac{1+\alpha_{BA}}{L}\right)(V_A - V_B)$$

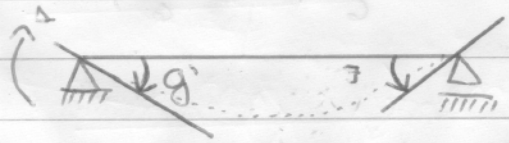
$$R_A = R_{AB} + \frac{(M_A - M_{AB})}{L} + \frac{(M_B - M_{BA})}{L}$$

$$R_B = R_{BA} - \frac{(M_A - M_{AB})}{L} - \frac{(M_B - M_{BA})}{L}$$

Calcular f, g, g' em função de β_{AB}, β_{BA}, α_{AB}, α_{BA}.

$$\beta_{AB} = \frac{g}{gg' - f^2} \qquad \beta_{BA} = \frac{g'}{gg' - f^2}$$

$$\alpha_{AB}\cdot\beta_{AB} = \alpha_{BA}\cdot\beta_{BA} = \frac{f}{gg' - f^2}$$



<u>Resolução pelo método dos deslocamentos</u>

Graus de Liberdade: φ_A φ_B

Eq de Equilíbrio: $M_A = -1$ $M_B = 0$

Caracterização das vigas: β_{AB}, β_{BA}, α_{AB}, α_{BA} $M_{AB} = 0$ $M_{BA} = 0$

Equações fundamentais: $M_A = \cancel{M_{AB}} + \beta_{AB}\varphi_A + \beta_{BA}\alpha_{BA}\varphi_B$

$M_B = \cancel{M_{BA}} + \beta_{AB}\alpha_{AB}\varphi_A + \beta_{BA}\varphi_B$

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Sistema global:

$m_A = -1 = \beta_{AB} \varphi_A + \beta_{BA} \cdot \alpha_{BA} \cdot \varphi_B$

$m_B = 0 = \beta_{AB} \alpha_{AB} \varphi_A + \beta_{BA} \varphi_B$

$$\varphi_A = \frac{\begin{vmatrix} -1 & \beta_{BA}\alpha_{BA} \\ 0 & \beta_{BA} \end{vmatrix}}{\beta_{AB}\beta_{BA} - (\beta_{AB}\alpha_{AB})^2} = \frac{-\beta_{BA}}{\beta_{AB}\beta_{BA} - (\alpha_{AB}\beta_{AB})^2}$$

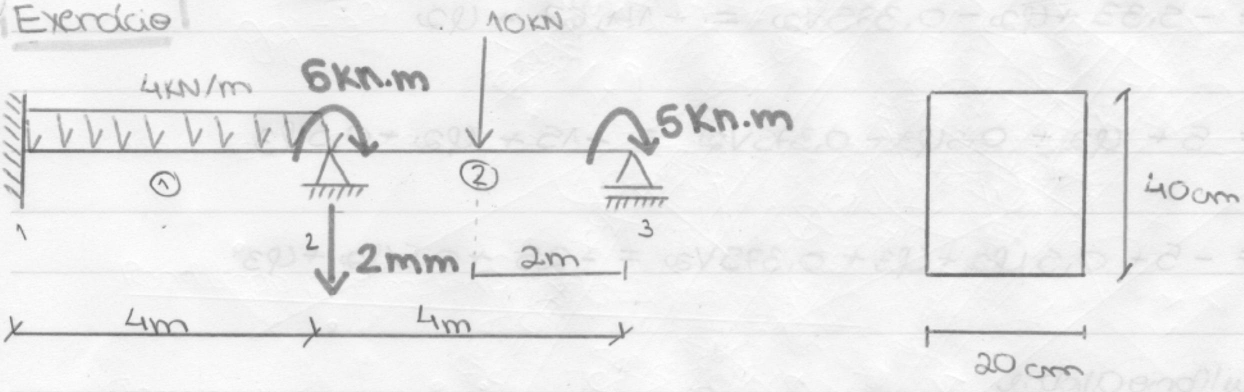
$$\varphi_B = \frac{\begin{vmatrix} \beta_{AB} & -1 \\ \beta_{AB}\alpha_{AB} & 0 \end{vmatrix}}{Det} = \frac{\beta_{AB} \cdot \alpha_{AB}}{Det}$$

$$g' = -\varphi_A = \frac{\beta_{BA}}{det} \qquad f = \varphi_B = \frac{\alpha_{AB}\beta_{AB}}{det}$$

$Det = \beta_{AB}\beta_{BA} - (\alpha_{AB}\beta_{AB})^2$

$\boxed{\text{FAZER O MESMO PARA } g}$

Exercício



$E = 25.000 \text{ MPa} \quad I = \frac{bh^3}{12} = 106666,7 \text{ cm}^4 \quad \{EI = 26667\}$

$V_2 = -2 \times 10^{-3} EI = -53,33$

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① Graus de Liberdade GDL : φ_2 φ_3

② Equações de Equilíbrio $M_B^{①} + M_A^{②} = -5$ $M_B^{②} = -5$

③ Caracterização das Vigas :

$\boxed{\text{Viga 1}}$

$L = 4$ $L' = 4$ $\beta = 1$ $\alpha = 1/2$

$M_{AB} = 5{,}33$ $M_{BA} = -5{,}33$

$\boxed{\text{Viga 2}}$ $L = 4$ $L' = 4$ $\beta = 1$ $\alpha = 0{,}5$

$$M_{AB} = P \cdot \frac{a b^2}{L^2} = 10 \cdot \frac{8}{16} = 5 \qquad M_{BA} = -5$$

④ Expressões fundamentais

- $M_A^{①} = 5{,}33 + \beta_{AB}^{1} \cancel{\varphi_A}^{0} + 1 \cdot \frac{1}{2} \cdot \varphi_2 + 1 \left(\frac{1 + 1/2}{4} \right) (0 - V_2)$

$M_A^{①} = 5{,}33 + 0{,}5\varphi_2 - 0{,}375 V_2 = 25{,}33 + 0{,}5\varphi_2$

- $M_B^{①} = -5{,}33 + \varphi_2 - 0{,}375 V_2 = +14{,}67 + \varphi_2$

- $M_A^{②} = 5 + \varphi_2 + 0{,}5\varphi_3 + 0{,}375 V_2 = -15 + \varphi_2 + 0{,}5\varphi_3$

- $M_B^{②} = -5 + 0{,}5\varphi_2 + \varphi_3 + 0{,}375 V_2 = -25 + 0{,}5\varphi_2 + \varphi_3$

⑤ Equilíbrio global

$M_B^{①} + M_A^{②} = -0{,}33 + 2\varphi_2 + 0{,}5\varphi_3 = -5$

$M_B^{②} = -25 + 0{,}5\varphi_2 + \varphi_3 = -5$

spiral

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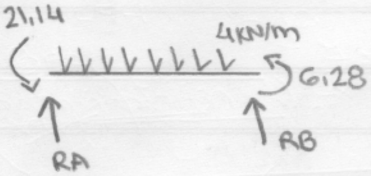
$$\begin{bmatrix} 2 & 0{,}5 \\ 0{,}5 & 1 \end{bmatrix} \begin{bmatrix} \varphi_2 \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} -4{,}67 \\ 20 \end{bmatrix}$$

$\varphi_2 = -8{,}38$
$\varphi_3 = 24{,}19$

$$\begin{cases} m_A^{①} = 21{,}14 \; \curvearrowleft \\ m_B^{①} = 6{,}28 \; \curvearrowleft \\ m_A^{②} = -11{,}28 \; \curvearrowright \\ m_B^{②} = -5 \; \curvearrowright \end{cases}$$

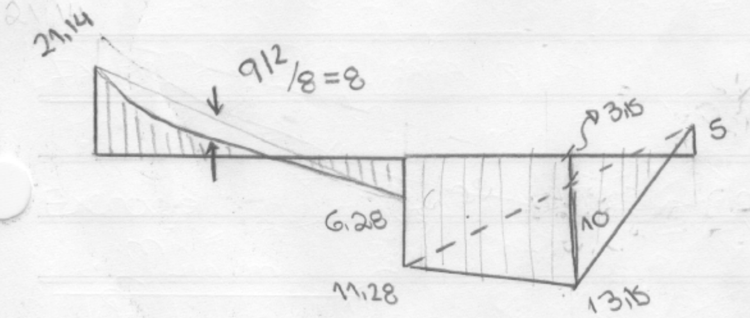
Diagramas

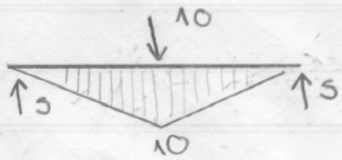


$RA = 8 + \dfrac{21{,}14 + 6{,}28}{4} = 14{,}85 \text{ kN}$

$RB = 1{,}15$

* Momento (kN.m)





*Cortante (kN)