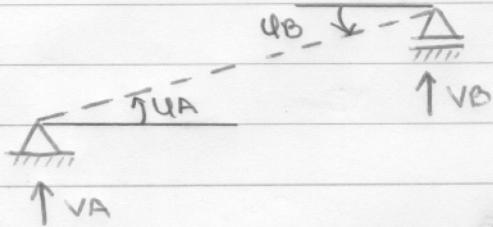


* Método dos deslocamentos

$$M_A = M_{AB} + \beta_{AB} \psi_A + \beta_{AB} \alpha_{AB} \psi_B$$

$$M_B = M_{BA} + \beta_{BA} \alpha_{BA} \psi_A + \beta_{BA} \psi_B$$

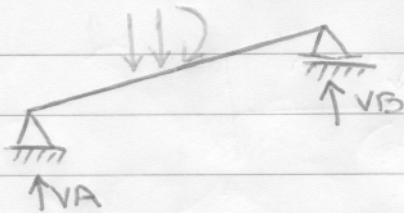


$$\psi_A = \frac{V_B - V_A}{L}$$

$$\psi_B = \frac{V_B - V_A}{L}$$

$$0 = \beta_{AB} \left(\psi_A - \left(\frac{V_B - V_A}{L} \right) \right) + \beta_{AB} \alpha_{AB} \left(\psi_B - \left(\frac{V_B - V_A}{L} \right) \right)$$

$$0 = \beta_{BA} \alpha_{BA} \left(\psi_A - \left(\frac{V_B - V_A}{L} \right) \right) + \beta_{BA} \left(\psi_B - \left(\frac{V_B - V_A}{L} \right) \right)$$



$$\begin{cases} -\psi_A = -a - m_A g' + m_B \beta + (V_A - V_B) & (I) \\ \psi_B = b - m_A \beta + m_B g - (V_A - V_B) \end{cases}$$

Multiplicando a expressão (I) por (-1) para facilitar os cálculos, temos:

$$\begin{bmatrix} +g' & -\beta \\ -\beta & g \end{bmatrix} \begin{bmatrix} m_A \\ m_B \end{bmatrix} = \begin{bmatrix} \psi_A + a + \left(\frac{V_A - V_B}{L} \right) \\ \psi_B - b + \left(\frac{V_A - V_B}{L} \right) \end{bmatrix}$$

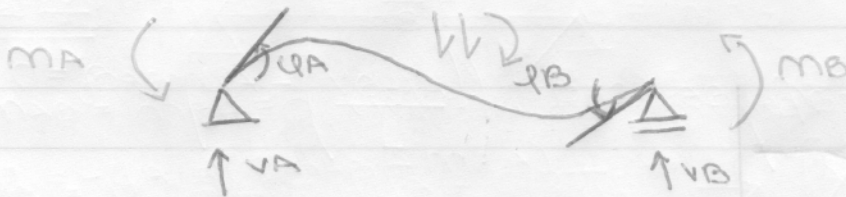
$$\begin{bmatrix} m_A \\ m_B \end{bmatrix} = \begin{bmatrix} g & \beta \\ \beta & g' \end{bmatrix} \begin{bmatrix} \psi_A + a + \left(\frac{V_A - V_B}{L} \right) \\ \psi_B - b + \left(\frac{V_A - V_B}{L} \right) \end{bmatrix}$$

Obs: $\begin{bmatrix} a & b \\ b & c \end{bmatrix}^{-1} = \begin{bmatrix} c & -b \\ -b & a \end{bmatrix} \cdot \frac{1}{ac - b^2}$

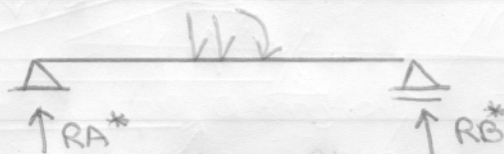
$$\begin{bmatrix} m_A \\ m_B \end{bmatrix} = \begin{bmatrix} g\psi_A + 3\psi_B + (ga - 3b) + (g+z)\left(\frac{VA-VB}{L}\right) \\ 3\psi_A + g\psi_B + (3a - g'b) + (g'+z')\left(\frac{VA-VB}{L}\right) \end{bmatrix} \cdot \frac{1}{gg' - z^2}$$

$$m_A = m_{AB} + \beta_{AB}\psi_A + \beta_{AB}\alpha_{AB}\psi_B + \beta_{AB}\left(\frac{1+\alpha_{AB}}{L}\right)(VA-VB)$$

$$m_B = m_{BA} + \beta_{BA}\alpha_{BA}\psi_A + \beta_{BA}\psi_B + \beta_{BA}\left(\frac{1+\alpha_{BA}}{L}\right)(VA-VB)$$

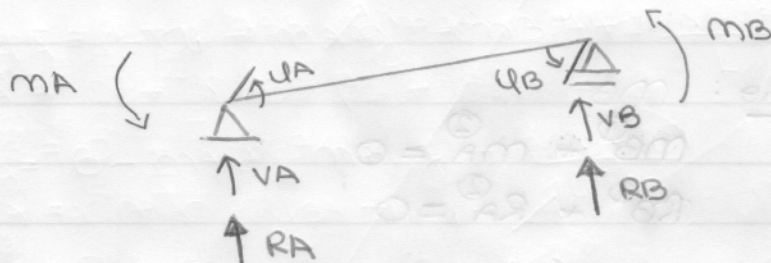


* Dado uma viga biapoiada



$$R_A = R_A^* + \frac{(m_A + m_B)}{L}$$

$$R_B = R_B^* + \frac{(m_A + m_B)}{L}$$

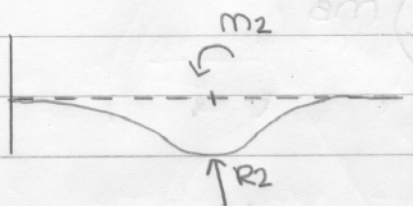
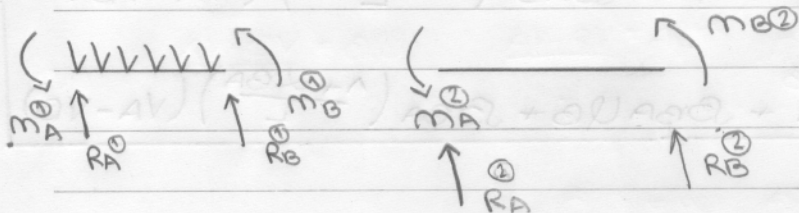
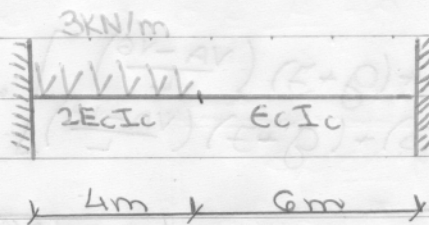


$$R_A = R_{AB} + \beta_{AB}\frac{(1+\alpha_{AB})}{L}\psi_A + \beta_{BA}\frac{(1+\alpha_{BA})}{L}\psi_B + K_{VAB}(VA-VB)$$

$$R_B = R_{BA} - \beta_{AB}\frac{(1+\alpha_{AB})}{L}\psi_A - \beta_{BA}\frac{(1+\alpha_{BA})}{L}\psi_B + K_{VAB}(VB-VA)$$

$$K_{VAB} = \frac{\beta_{AB}(1+\alpha_{AB}) + \beta_{BA}(1+\alpha_{BA})}{L^2}$$

Exemplo



$$m_2 = m_B^{(1)} + m_A^{(2)}$$

$$R_2 = R_B^{(1)} + R_A^{(2)}$$

• Parte 1 Grau de liberdade GDL: ψ_2 e v_2

• Equações de equilíbrio:

$$m_B^{(1)} + m_A^{(2)} = 0$$

$$R_B^{(1)} + R_A^{(2)} = 0$$

• Parte 2 Caracterização das vigas

Viga 1 $L=4$ $L'=2$ $\beta=2$ $\alpha=0.5$

Viga 2 $L=6$ $L'=6$ $\beta=0.667$ $\alpha=0.5$

E1 / 50 / 22

2 / 3 / 2

• Passo 3 Equações fundamentais

$$M_A^{\textcircled{1}} = 4 + 2 \cancel{q_1}^0 + 4q_2 + \frac{3}{4} (v_1^0 - v_2)$$

$$M_B^{\textcircled{1}} = -4 + \cancel{q_1}^0 + 2q_2 + \frac{3}{4} (v_1^0 - v_2)$$

$$R_A^{\textcircled{1}} = 6 + \frac{3}{4} \cancel{q_1}^0 + \frac{3}{4} q_2 + \frac{3}{8} (v_1^0 - v_2)$$

$$R_B^{\textcircled{1}} = 6 - \frac{3}{4} \cancel{q_1}^0 - \frac{3}{4} q_2 - \frac{3}{8} (v_1^0 - v_2)$$

$$M_A^{\textcircled{2}} = \frac{2}{3} q_2 + \frac{1}{3} \cancel{q_3}^0 + \frac{1}{6} (v_2 - v_3^0)$$

$$M_B^{\textcircled{2}} = \frac{1}{3} q_2 + \frac{2}{3} \cancel{q_3}^0 + \frac{1}{6} (v_2 - v_3^0)$$

$$R_A^{\textcircled{2}} = \frac{1}{6} q_2 + \frac{1}{6} \cancel{q_3}^0 + \frac{1}{18} (v_2 - v_3^0)$$

$$R_B^{\textcircled{2}} = -\frac{1}{6} q_2 - \frac{1}{6} \cancel{q_3}^0 - \frac{1}{18} (v_2 - v_3^0)$$

• Passo 4 Eq. de Equilíbrio

$$M_B^{\textcircled{1}} + M_A^{\textcircled{2}} = -4 + \frac{8}{3} q_2 + \left(-\frac{3}{4} + \frac{1}{6}\right) v_2 = 0$$

$$R_B^{\textcircled{1}} + R_A^{\textcircled{2}} = 6 + \left(\frac{1}{6} - \frac{3}{4}\right) q_2 + \left(\frac{3}{8} + \frac{1}{18}\right) v_2 = 0$$

$$\begin{bmatrix} 2,667 & -0,5833 \\ -0,5833 & 0,4306 \end{bmatrix} \begin{bmatrix} q_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

$$q_2 = 2,2$$

$$v_2 = -16,9$$