10/04/13

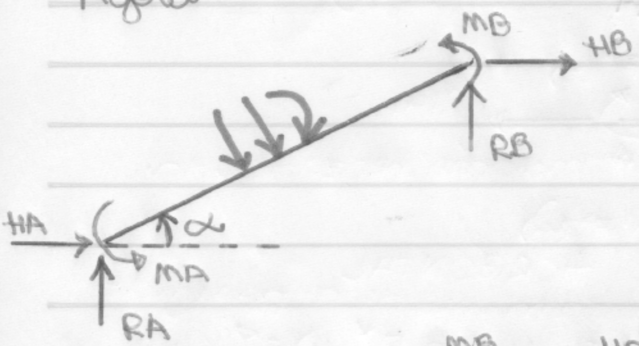
Método dos deslocamentos aplicado a viga inclinada

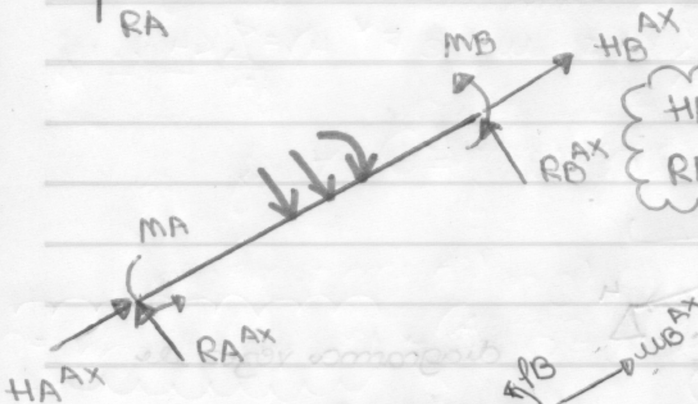
$$[F] = [F]^{ep} + [K][u]$$

$\sim$ β_{AB}; β_{BA}; α_{AB}; α_{BA}; $\frac{EA}{L E_c J_c}$; M_{AB}; M_{BA}; R_{AB}; R_{BA}; H_{AB}; H_{BA}

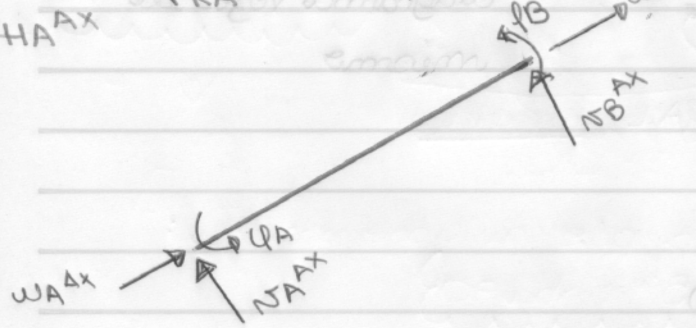
já sabidos

Agora:





$$HA = HA^{AX}\cos\alpha - RA^{AX}\,\text{sen}\,\alpha$$
$$RA = HA^{AX}\,\text{sen}\,\alpha + RA^{AX}\cos\alpha$$



$$uA^{AX} = uA\cos\alpha + vA\,\text{sen}\,\alpha$$
$$vA^{AX} = -uA\,\text{sen}\,\alpha + vA\cos\alpha$$

$$[F]_{AX} = [F]^{EP}_{AX} + [K]_{AX} \cdot [u]_{AX}$$

10/04/13

$$
\begin{bmatrix} H_A \\ R_A \\ M_A \\ H_B \\ R_B \\ M_B \end{bmatrix}
=
\underbrace{\left[\begin{array}{ccc|ccc}
\cos\alpha & -\operatorname{sen}\alpha & 0 & 0 & 0 & 0 \\
\operatorname{sen}\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\ \hline
0 & 0 & 0 & \cos\alpha & -\operatorname{sen}\alpha & 0 \\
0 & 0 & 0 & \operatorname{sen}\alpha & \cos\alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]}_{R^T}
\begin{bmatrix} H_A^{AX} \\ R_A^{AX} \\ M_A \\ H_B^{AX} \\ R_B^{AX} \\ M_B \end{bmatrix}
$$

$$
\begin{bmatrix} u_A^{AX} \\ v_A^{AX} \\ \varphi_A \\ u_B^{AX} \\ v_B^{AX} \\ \varphi_B \end{bmatrix}
=
\underbrace{\left[\begin{array}{ccc|ccc}
\cos\alpha & \operatorname{sen}\alpha & 0 & & & \\
-\operatorname{sen}\alpha & \cos\alpha & 0 & & 0 & \\
0 & 0 & 1 & & & \\ \hline
 & & & \cos\alpha & \operatorname{sen}\alpha & 0 \\
 & 0 & & -\operatorname{sen}\alpha & \cos\alpha & 0 \\
 & & & 0 & 0 & 1
\end{array}\right]}_{R}
\begin{bmatrix} u_A \\ v_A \\ \varphi_A \\ u_B \\ v_B \\ \varphi_B \end{bmatrix}
$$

$$\begin{cases} [F] = [R]^T . [F]_{AX} \\ [u]_{AX} = [R][u] \end{cases}$$

$\rightarrow [R].[R]^T = [I]$

$$\begin{cases} [F]_{AX} = [R].[F] \\ [u] = [R]^T . [u]_{AX} \end{cases}$$

Então:

$$[R]^T . [F]_{AX} = [R]^T . [F]^{ER}_{AX} + [R]^T . [R]_{AX} [u]_{AX}$$

$$[F] = [R]^T . [F]^{EP}_{AX} + [R]^T [K]_{AX} . [u_{AX}]$$

$$[F] = \underbrace{[R]^T . [F]^{EP}_{AX}}_{[F]^{EP}} + \underbrace{[R]^T [K]_{AX} [R]}_{[K]} [u]$$

matriz simétrica

$[K]^T = [K]$

10/04/13

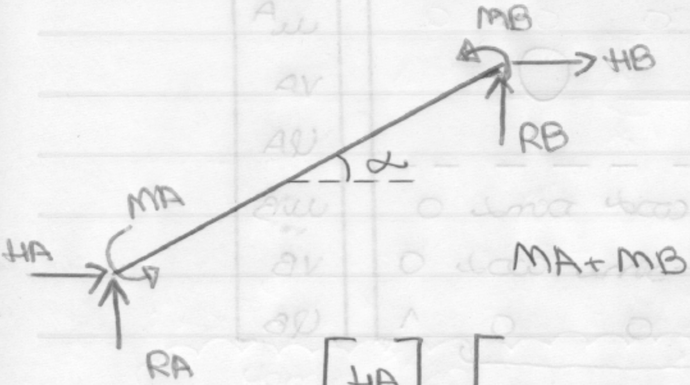
Propriedades da Matriz de Rigidez

① O sistema de forças obtidas por [K][u] é um sistema em equilíbrio ∀[u]

② caso [u] é um deslocamento de corpo rígido → [K][u]=0

③ $[F]^{EP}$ estão em equilíbrio com as forças externas aplicadas

PROPRIEDADE DA MATRIZ [K]



Equilíbrio se: $HA + HB = 0$

$RA + RB = 0$

$\sum M_A^{\circlearrowright} = 0$

$MA + MB + RB \cdot L \cdot \cos\alpha - HB \cdot L \cdot \operatorname{sen}\alpha = 0$

$$\begin{bmatrix} HA \\ RA \\ MA \\ HB \\ RB \\ MB \end{bmatrix} = \begin{bmatrix} & & \\ & \cdots & \\ & & \end{bmatrix} \begin{bmatrix} uA \\ vA \\ \varphi A \\ uB \\ vB \\ \varphi B \end{bmatrix}$$

$$HA = \sum_{j=1}^{6} K_{1j} \cdot u_j \qquad HB = \sum_{j=1}^{6} K_{4j} \cdot u_j$$

$$HA + HB = \sum_{j=1}^{6} (K_{1j} + K_{4j}) \cdot u_j = 0 \quad \forall u_j$$

1ª Linha + 4ª Linha = 0 (Termo a termo)

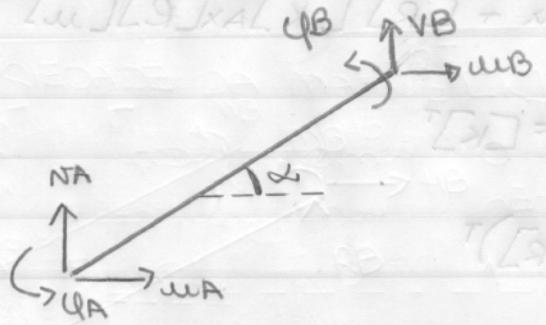
10 04 13

Análogo: $R_A + R_B = 0$, $\forall v \rightarrow$ $\boxed{2^a \text{ Linha} + 5^a \text{ Linha} = 0}$

$M_A + M_R + R_B . L . \cos\alpha - H_B . L \operatorname{sen}\alpha = 0$

$\boxed{3^a l + 6^a l + 5^a l . L . \cos\alpha - 4^a l . L . \cos\alpha = 0}$

Movimento de Corpo Rígido



translação em x: $[u]^T = [t \; 0 \; 0 \; t \; 0 \; 0]$

translação em y: $[u]^T = [0 \; t \; 0 \; 0 \; t \; 0]$

giro φA em torno de A: $[u] = [0 \;\; 0 \;\; \varphi_A \;\; -\varphi_A L \operatorname{sen}\alpha \;\; \varphi_A L \cos\alpha \;\; \varphi_A]$

translação em x: $[F] = [K][u]$

$$F_i = \sum K_{ij} \, u_j$$

$$F_i = K_{i1} . t + K_{i4} . t$$

$$F_i = (K_{i1} + K_{i4}) t$$

$$\downarrow$$

$$K_{i1} + K_{i4} = 0 \quad \forall i$$

$\boxed{1^a \text{ Coluna} + 4^a \text{ Coluna} = 0}$ (Termo a Termo)

10/04/13

Por analogia:

translação em y: 2ª coluna + 5ª coluna = 0

Momento em A: 3ª col + 6ª col − L sen α 4ª col + L cos α 5ª C = 0

Voltando:

$$[F] = [R]^T . [F]^{EP}_{Ax} + [R]^T [K]_{Ax} [R] [u]$$

Como $[K]$ é simétrica → $[K] = [K]^T$

$$[K]^T = \left([R]^T . [K]_{Ax} . [R]\right)^T$$

$\boxed{\text{Obs: } [AB]^T = B^T . A^T}$

$$[K]^T = [R]^T . [K]^T_{Ax} . [R]^{T^T} = [R]^T . [K]^T_{Ax} . [R]$$

$$\downarrow$$

simétrica

$$[K]^T = [R]^T . [K]_{Ax} . [R]$$

$$[K]^T = [K]$$