$$[F]^{AX} = [F_{ep}]^{AX} + [K]^{AX}\,[u]^{AX}$$

$$[R]^T [F]^{AX} = [R]^T [F_{ep}]^{AX} + [R]^T . [K]^{AX} . [u]^{AX}$$

$$[F] = \underbrace{[R]^T . [F_{ep}]^{AX}} + \underbrace{[R]^T . [K]^{AX} . [R]}[u]$$

$$[F] = [F_{ep}] + [K][u]$$

$$[R] = \begin{bmatrix} \cos\alpha & \operatorname{sen}\alpha & 0 & 0 \\ -\operatorname{sen}\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \operatorname{sen}\alpha \\ 0 & 0 & -\operatorname{sen}\alpha & \cos\alpha \end{bmatrix}$$

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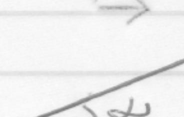
Revisão aula passada:

$$H_A^{AX} = H_{AB}^{AX} + \frac{E_A}{L E_c I_c}\left(u_A^{AX} - u_B^{AX}\right)$$

$$R_A^{AX} = R_{AB}^{AX}$$

$$H_B^{AX} = H_{BA}^{AX} + \frac{E_A}{L E_c I_c}\left(u_B^{AX} - u_A^{AX}\right)$$

$$R_B^{AX} = R_{BA}^{AX}$$



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$$[F] = [R]^T \cdot (Fep)^{AX} + [R]^T \cdot [K]^{AX} [R][w]$$

$$[K]^{AX} = \frac{EA}{L E_c I_c} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K] = \frac{EA}{L E_c I_c} \begin{bmatrix} \cos^2 & \cos\,\text{sen} & -\cos^2 & -\cos\,\text{sen} \\ \cos\,\text{sen} & \text{sen}^2 & -\cos\,\text{sen} & -\text{sen}^2 \\ -\cos^2 & -\cos\,\text{sen} & \cos^2 & \cos\,\text{sen} \\ -\cos\,\text{sen} & -\text{sen}^2 & \cos\,\text{sen} & \text{sen}^2 \end{bmatrix}$$

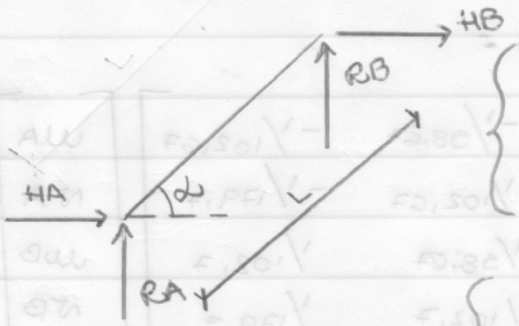
$\cos = \cos\alpha$

$\text{sen} = \text{sen}\,\alpha$

Propriedades da Matriz [K]

* $[F] = [K][w]$

[F] é um sistema de forças em equilíbrio



$$\begin{cases} HA + HB = 0 \\ RA + RB = 0 \\ RB \cdot L\cos\alpha - HB \cdot L\,\text{sen}\,\alpha = 0 \end{cases}$$

$$\begin{cases} 1^a L + 3^a L = 0 \\ 2^a L + 4^a L = 0 \\ 4^a L \cdot \cos\alpha + 3^a L \cdot \text{sen}\,\alpha = 0 \end{cases}$$

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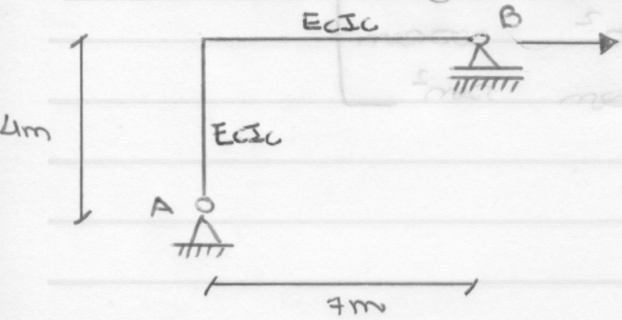
* Movimento de corpo rígido

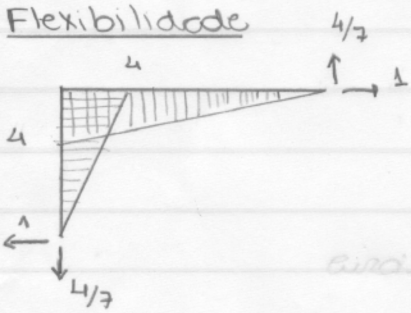
$$[K]\begin{bmatrix} u \\ 0 \\ u \\ 0 \end{bmatrix} = [0] \qquad [K]\begin{bmatrix} 0 \\ v \\ 0 \\ v \end{bmatrix} = [0] \qquad [K]\begin{bmatrix} 0 \\ 0 \\ -\omega L\,\mathrm{sen}\,\alpha \\ \omega L\,\mathrm{sen}\,\alpha \end{bmatrix} = [0]$$

Dedução da matriz de rigidez a partir da flexibilidade



Cálculo da rigidez do elemento "treliça" entre A e B

Flexibilidade



$$u_B = H_B \cdot \left(\frac{1}{3} \times 4 \times 4 \times 4 + \frac{1}{3} \times 7 \times 4 \times 4\right)$$

$$u_B = H_B \cdot 58{,}67 \rightarrow u_B \cdot \frac{1}{58{,}67} = H_B$$

$$\begin{bmatrix} H_A \\ R_A \\ H_B \\ R_B \end{bmatrix} = \begin{bmatrix} 1/58{,}67 & 1/102{,}67 & -1/58{,}67 & -1/102{,}67 \\ 1/102{,}7 & 1/179{,}7 & -1/102{,}67 & -1/179{,}7 \\ -1/58{,}67 & -1/102{,}7 & 1/58{,}67 & 1/102{,}7 \\ -1/102{,}7 & -1/179{,}7 & 1/102{,}7 & 1/179{,}7 \end{bmatrix} \begin{bmatrix} u_A \\ v_A \\ u_B \\ v_B \end{bmatrix}$$

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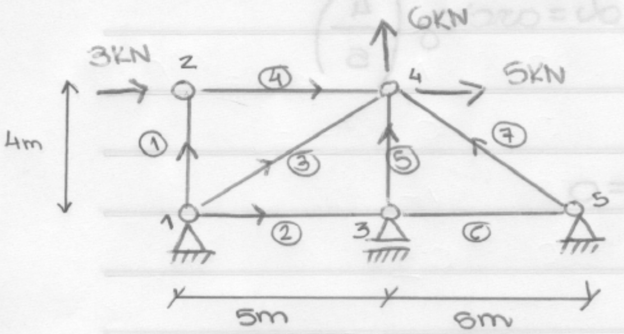
Resolução de uma treliça pelo método dos deslocamentos



barra	A
1	10^{-4}
6-2	$2\cdot10^{-4}$
6-3	$3\cdot10^{-4}$
7-4	10^{-3}

⑦ $\Delta T = 20°C$ $\quad E = 205000\ MPa$

⑤ $\delta l = -3mm$ $\quad \alpha = 10^{-5}/°C$

$V_1 = -2mm$ $\quad E_cI_c = E\cdot 10^{-3}$

① Graus de Liberdade: $u2, v2, u3, u4, v4,$

② Eq. de Equilíbrio

$u2 \quad H_B^{①} + H_A^{④} = 3$

$v2 \quad R_B^{①} + R_A^{④} = 0$

$u3 \quad H_B^{②} + H_A^{⑤} + H_A^{⑥} = 0$

$u4 \quad H_B^{③} + H_B^{④} + H_B^{⑤} + H_B^{⑦} = 5$

$v4 \quad R_B^{②} + R_B^{④} + R_B^{⑦} + R_B^{⑤} = 6$

③ caracterização das Vigas

Barra 1 - $L = 4$ $\quad \frac{EA}{E_cI_c} = 0,1$ $\quad \alpha = \pi/2$

$H_{AB} = H_{BA} = R_{AB} = R_{BA} = 0$

Barra 2 - $L = 5$ $\quad \frac{EA}{E_cI_c} = 0,2$ $\quad \alpha = 0$

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Barra3 - $L = \sqrt{41}$ $\quad \dfrac{EA}{E_cI_c} = 0,3 \quad \alpha = \text{arctg}\left(\dfrac{4}{5}\right)$

Barra4 - $L = 5 \quad \dfrac{EA}{E_cI_c} = 1 \quad \alpha = 0$

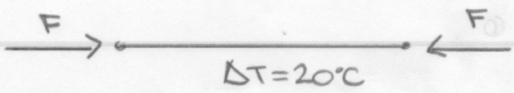
Barra5 $\quad L = 4 \quad \dfrac{EA}{E_cI_c} = 0,2 \quad \alpha = \dfrac{\pi}{2}$

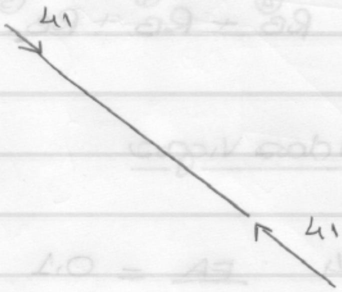
$$\delta l = F\,\frac{L}{EA} \qquad F = 0,003 \cdot \frac{205 \cdot 10^{6}}{4} \cdot 2\,10^{-4} = 37,5\text{ KN}$$

$H_{AB} = 0 \quad R_{AB} = -37,5 \quad H_{BA} = 0 \quad R_{BA} = 37,5$

Barra6 $\quad L = 5 \quad \dfrac{EA}{E_cI_c} = 0,3 \quad \alpha = 0$

Barra7 $\quad L = \sqrt{41} \quad \dfrac{EA}{E_cI_c} = 1 \quad \alpha = \text{arctg}\,(-5,4)$





$$\delta l = F \cdot \frac{L}{EA} = \alpha \cdot 20 \cdot L$$

$F = \alpha \cdot 20 \cdot EA$

$= 10^{-5} \cdot 20 \cdot 205 \cdot 10^{6} \cdot 10^{-3}$

$= 41\text{KN}$

$H_{AB} = -\dfrac{41 \cdot 5}{\sqrt{41}} = -32,02 \quad R_{AB} = \dfrac{41 \cdot 4}{\sqrt{41}} = 25,6 \quad H_{BA} = -H_{AB}$

$R_{BA} = -R_{AB}$

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④ Equações fundamentais

$$\left.\begin{aligned}
H_A^{(1)} &= 0 \\
R_A^{(1)} &= 0{,}025 \cdot (v_1 - v_2) \\
H_B^{(1)} &= 0 \\
R_B^{(1)} &= 0{,}025\,(v_2 - v_1)
\end{aligned}\right\} \text{Barra 1}$$

$$\left.\begin{aligned}
H_A^{(3)} &= 0{,}047\,(0{,}61\,w_1 + 0{,}486\,v_1 - 0{,}61\,w_4 - 0{,}486\,v_4) \\
R_A^{(3)} &= 0{,}047\,(0{,}486\,w_1 + 0{,}39\,v_1 - 0{,}486\,w_4 - 0{,}39\,v_4) \\
H_B^{(3)} &= -H_A^{(3)} \\
R_B^{(3)} &= -R_A^{(3)}
\end{aligned}\right\} \text{Barra 3}$$