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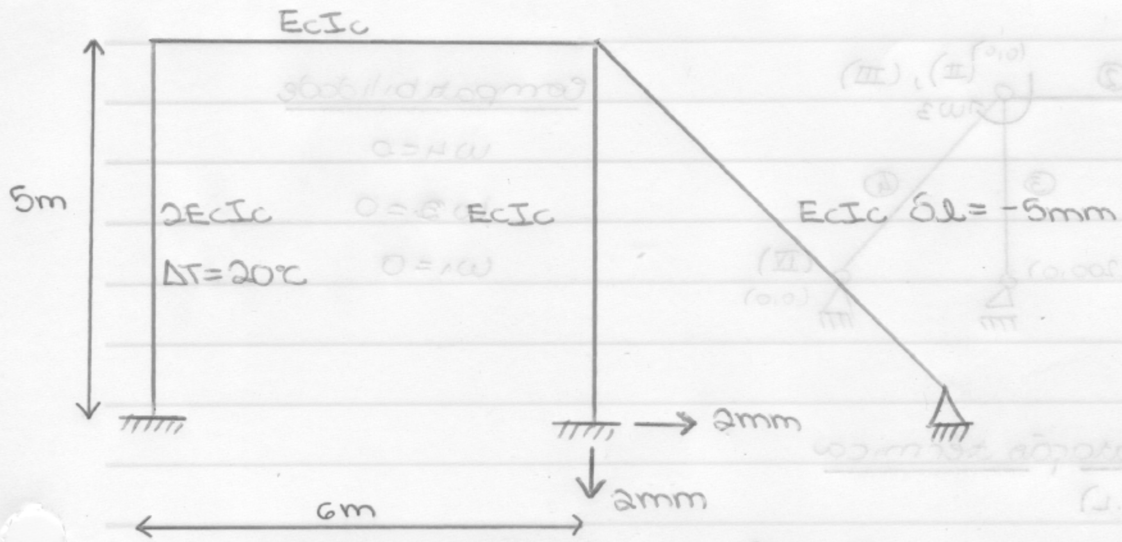
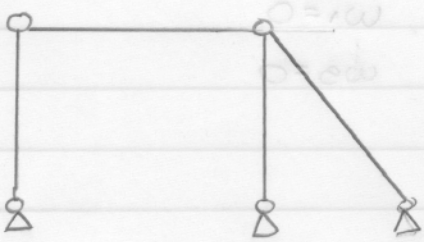
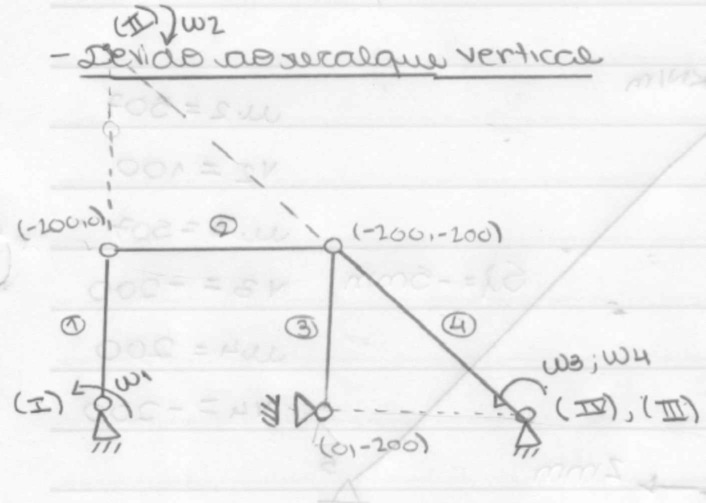




Utilização de cadeia cinemática para calcular os deslocamentos

- Devido ao recalque vertical



Compatibilidade

$\omega_1 . 5 = \omega_2 . 6$

$\omega_3 = \omega_4$

$\omega_3 . 5 = \omega_2 . 6$

deslocamento vertical 2mm

$\bar{V}_3 = -2mm$ ou 200

$V_3 = -200$ $\quad \omega_3 = 40$

↳ $V = \bar{V}_3 . E_cI_c$

Resolução

$\omega_3 = 40 \quad \omega_4 = 40$

$\omega_2 = \frac{40.5}{6} = 33,33 \quad \omega_1 = 40$

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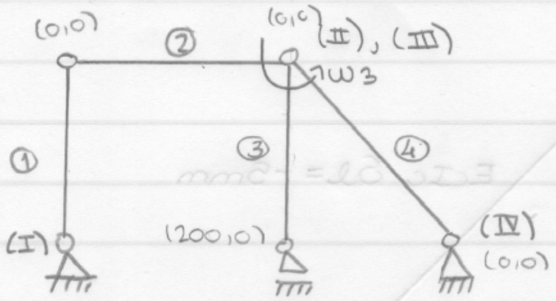
Devido ao recalque horizontal



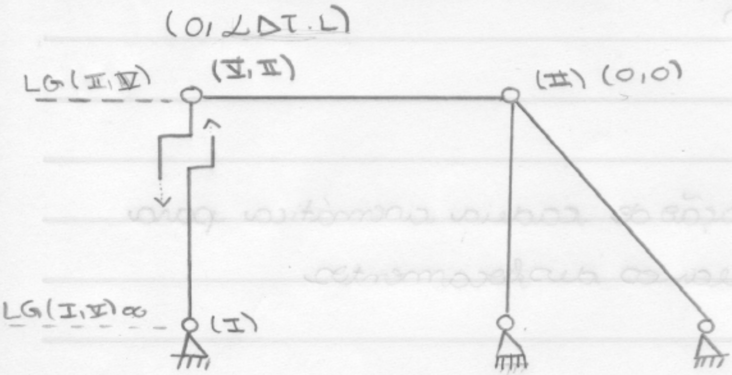
Compatibilidade

$\omega_4 = 0$

$\omega_2 = 0$

$\omega_1 = 0$

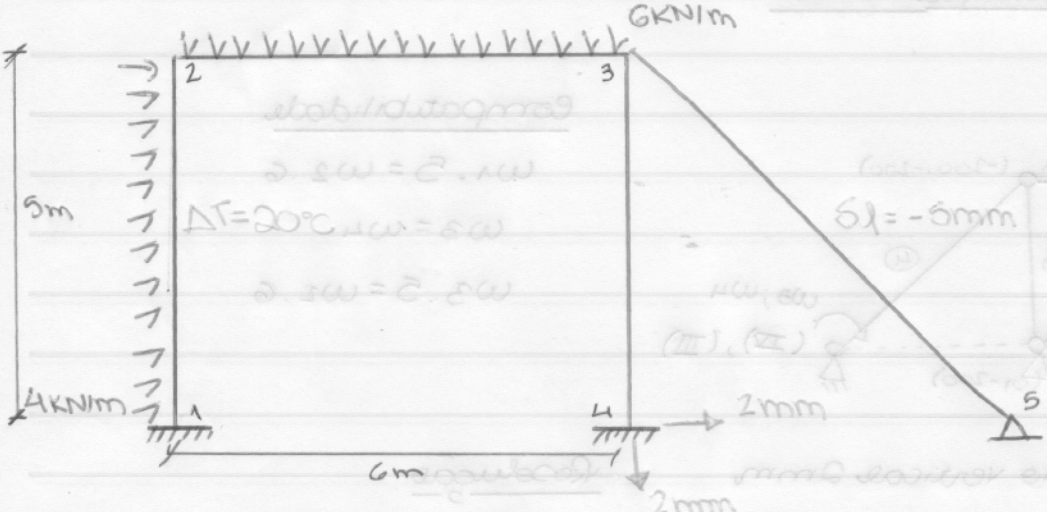
Devido à dilatação térmica



Compatibilidade

$\omega_1 = 0$

$\omega_5 = 0$



$w_2 = 507$

$v_2 = 100$

$w_3 = 507$

$v_3 = -200$

$w_4 = 200$

$v_4 = -200$

Caracterização das barras:

Barra 1

$L_1 = 5$ $\quad L'_1 = 2,5$ $\quad \beta = 1,6$ $\quad w_1 = 0$ $\quad v_1 = 0$

$w_2 = 507$ $\quad v_2 = 100$

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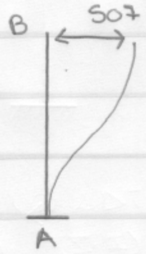
$$MA = \beta AB \, \frac{(1+\alpha AB)}{L} (VA - VB) \qquad MAB = 1{,}6 \cdot \frac{(1{,}5)}{5} \cdot (507) = 243{,}36$$

$$MB = \beta AB \cdot \frac{(1+\alpha BA)}{L} \cdot (VA - VB) \qquad MBA = 1{,}6 \cdot \frac{1{,}5}{5} (507) = 243{,}36$$



$VB = -507$

$$\overset{①}{M}AB = 8{,}33 + 243{,}36 = 251{,}69$$

$$\overset{①}{M}BA = -8{,}33 + 243{,}36 = 235{,}03$$

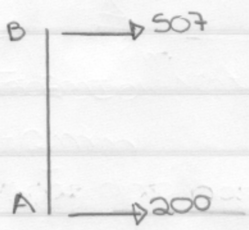
Barra 2

$L = 6 \quad L' = 6 \quad \beta = 0{,}667$

$$MAB = \frac{PL^2}{12} + 0{,}667 \cdot \frac{1{,}5}{6} (100 + 200) = 68$$

$$MBA = -\frac{PL^2}{2} + 0{,}667 \cdot \frac{1{,}5}{6} (100 + 200) = 32$$

Barra 3



$L = 5 \quad L' = 5 \quad \beta = 0{,}8$

$$MAB = \frac{0{,}8 \cdot (1{,}5)}{5} (-200 + 507) = 73{,}68$$

$MBA = 73{,}68$

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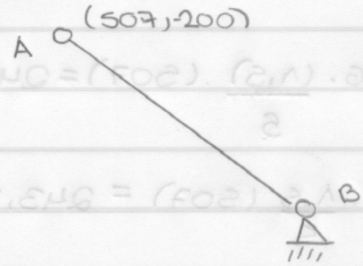
Barra 4



$$\beta = \frac{4}{5\sqrt{2}} = 0{,}567$$

$$V_A^{Ax} = \frac{507 \cdot \sqrt{2}}{2} - \frac{200\sqrt{2}}{2} = 217{,}08$$

$$M_{AB} = 0{,}567 \cdot \frac{(1{,}5)}{5\sqrt{2}} (217{,}08) = 26{,}06$$

$$M_{BA} = 26{,}06$$

① $\beta = 1{,}6$ $M_{AB} = 251{,}69$ $M_{BA} = 235{,}03$

② $\beta = 0{,}667$ $M_{AB} = 68$ $M_{BA} = 32$

③ $\beta = 0{,}8$ $M_{AB} = 73{,}68$ $M_{BA} = 73{,}68$

④ $\beta = 0{,}567$ $M_{AB} = 26{,}06$ $M_{BA} = 26{,}06$

GDL	Eq Eq
φ_2	$M_B^{(1)} + M_A^{(2)} = 0$
φ_3	$M_B^{(2)} + M_B^{(3)} + M_A^{(4)} = 0$
φ_5	$M_B^{(4)} = 0$

Eq. fund.:

$M_A^{(1)} = 251{,}69 + 0{,}8\,\varphi_2$

$M_B^{(1)} = 235{,}03 + 1{,}6\,\varphi_2$

$M_A^{(2)} = 68 + 0{,}667\,\varphi_2 + 0{,}333\,\varphi_3$

$M_B^{(2)} = 32 + 0{,}333\,\varphi_2 + 0{,}667\,\varphi_3$

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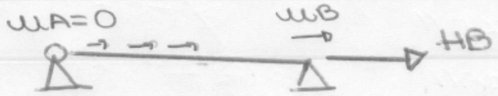
$$M_A^{(3)} = 73{,}68 + 0{,}4\,\varphi_3$$
$$M_B^{(3)} = 73{,}68 + 0{,}8\,\varphi_3$$
$$M_A^{(4)} = 26{,}06 + 0{,}567\,\varphi_3 + 0{,}283\,\varphi_5$$
$$M_B^{(4)} = 26{,}06 + 0{,}283\,\varphi_3 + 0{,}567\,\varphi_5$$

$$\begin{bmatrix} 2{,}267 & 0{,}333 & 0 \\ 0{,}333 & 2{,}033 & 0{,}283 \\ 0 & 0{,}283 & 0{,}567 \end{bmatrix} \begin{bmatrix} \varphi_2 \\ \varphi_3 \\ \varphi_5 \end{bmatrix} = \begin{bmatrix} -251{,}69 \\ \\ \\ \end{bmatrix}$$

Resolução de Treliças pelo Método dos deslocamentos

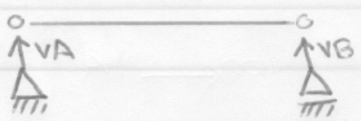
Barra horizontal



$$HB = \frac{EA}{L\,E_cI_c}\,u_B + HBA$$

Generalizando:

$$\boxed{\begin{aligned} HA &= HAB + \frac{EA}{L\,E_cI_c}(u_A - u_B) \\ HB &= HBA + \frac{EA}{L\,E_cI_c}(u_B - u_A) \end{aligned}}$$

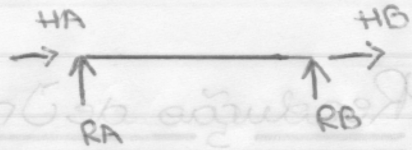


$$\boxed{\begin{aligned} RA &= RAB + 0\,(VA - VB) \\ RB &= RBA + 0\,(VA - VB) \end{aligned}}$$

$$\begin{bmatrix} HA \\ RA \\ HB \\ RB \end{bmatrix} = \begin{bmatrix} HAB \\ RAB \\ HBA \\ RBA \end{bmatrix} + \begin{bmatrix} \frac{EA}{LE_cI_c} & 0 & -EA/LE_cI_c & 0 \\ 0 & 0 & 0 & 0 \\ -EA/LE_cI_c & 0 & EA/LE_cI_c & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} uA \\ VA \\ uB \\ VB \end{bmatrix}$$

Propriedades da matriz rigidez

$$\begin{bmatrix} HA \\ RA \\ HB \\ RB \end{bmatrix} = \frac{EA}{L.E_cI_c} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} uA \\ VA \\ uB \\ VB \end{bmatrix}$$



Equações de Equilíbrio

$HA + HB = 0$

$RA + RB = 0$

$RB = 0$

Movimento de corpo rígido

$$[K] \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = [0]$$

$$[K] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = [0] \qquad [K] \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = [0]$$

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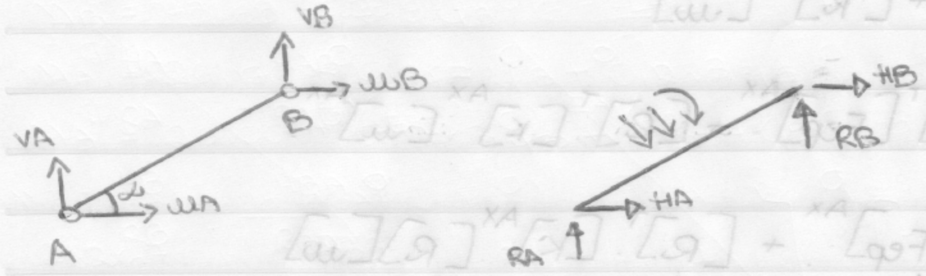
Cálculo da rigidez de elemento de treliça inclinada



$$\begin{cases} H_A^{AX} = H_{AB}^{AX} + \dfrac{EA}{L_{EcIc}} \left(u_A^{AX} - u_B^{AX}\right) \\ H_B^{AX} = H_{BA}^{AX} + \dfrac{EA}{L_{EcIc}} \left(u_B^{AX} - u_A^{AX}\right) \\ R_A^{AX} = R_{AB}^{AX} \\ R_B^{AX} = R_{BA}^{AX} \end{cases}$$

$$H_A = H_A^{AX}\cos\alpha - R_A^{AX}\operatorname{sen}\alpha$$
$$R_A = H_A^{AX}\cos\alpha + R_A^{AX}\operatorname{sen}\alpha$$
$$H_B = H_B^{AX}\cos\alpha - R_B^{AX}\operatorname{sen}\alpha$$
$$R_B = H_B^{AX}\operatorname{sen}\alpha + R_B^{AX}\cos\alpha$$

$$\begin{bmatrix} H_A \\ R_A \\ H_B \\ R_B \end{bmatrix} = [R]^T \begin{bmatrix} H_A^{AX} \\ R_A^{AX} \\ H_B^{AX} \\ R_B^{AX} \end{bmatrix}$$

$$u_A^{AX} = u_A\cos\alpha + V_A\operatorname{sen}\alpha$$
$$V_A^{AX} = -u_A\operatorname{sen}\alpha + V_A\cos\alpha$$
$$u_B^{AX} = u_B\cos\alpha + V_B\operatorname{sen}\alpha$$
$$V_B^{AX} = -u_B\operatorname{sen}\alpha + V_B\cos\alpha$$

$$\begin{bmatrix} u_A^{AX} \\ V_A^{AX} \\ u_B^{AX} \\ V_B^{AX} \end{bmatrix} = [R] \begin{bmatrix} u_A \\ V_A \\ u_B \\ V_B \end{bmatrix}$$

$$[F]^{AX} = [F_{ep}]^{AX} + [K]^{AX} [u]^{AX}$$

$$[R]^T [F]^{AX} = [R]^T [F_{ep}]^{AX} + [R]^T [K]^{AX} [u]^{AX}$$

$$[F] = \underbrace{[R]^T [F_{ep}]^{AX}} + \underbrace{[R]^T [K]^{AX} [R]}[u]$$

$$[F] = [F_{ep}] + [K][u]$$

$$[R] = \begin{bmatrix} \cos\alpha & \text{sen}\,\alpha & 0 & 0 \\ -\text{sen}\,\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \text{sen}\,\alpha \\ 0 & 0 & -\text{sen}\,\alpha & \cos\alpha \end{bmatrix}$$

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Revisão aula passada:

$$H_A^{AX} = H_{AB}^{AX} + \frac{EA}{L E_c I_c} \left(u_A^{AX} - u_B^{AX} \right)$$

$$R_A^{AX} = R_{AB}^{AX}$$

$$H_B^{AX} = H_{BA}^{AX} + \frac{EA}{L E_c I_c} \left(u_B^{AX} - u_A^{AX} \right)$$

$$R_B^{AX} = R_{BA}^{AX}$$