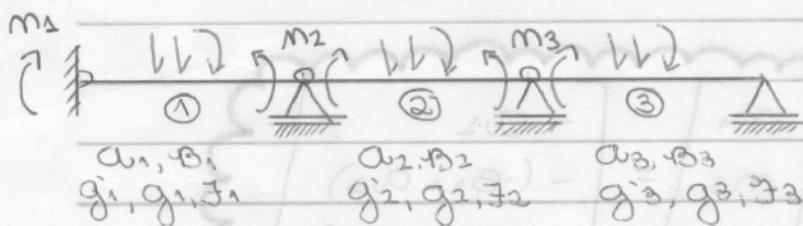


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Resolução de vigas contínuas pelo método das forças



Ex hiperestáticas

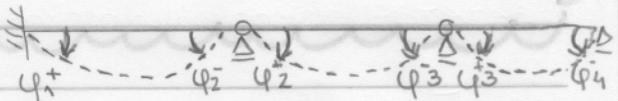
(aproveitam 3 nés)

→ surge M_1, M_2, M_3 Equações de compatibilidade

$$\varphi_1^+ = 0 \quad (m_1)$$

$$\varphi_2^- + \varphi_2^+ = 0 \quad (m_2)$$

$$\varphi_3^- + \varphi_3^+ = 0 \quad (m_3)$$



Viga 1: $(\overset{m_1}{\underset{\Delta-R}{\varphi_1^+}} \rightarrow \overset{m_2}{\underset{\Delta-R}{\varphi_2^+}} \rightarrow \overset{m_3}{\underset{\Delta-R}{\varphi_3^+}})$

* $\varphi_1^+ = a_{11} + m_1 g_1 + m_2 f_1$

$\varphi_2^- = b_{21} + m_2 f_1 + m_3 g_1$

Viga 2

* $\varphi_2^+ = a_{22} + m_2 g_2 + m_3 f_2$

$\varphi_3^- = b_{32} + m_3 f_2 + m_4 g_2$

Viga 3:

* $\varphi_3^+ = a_{33} + m_3 g_3 + m_4 f_3$

$\varphi_4^- = b_{43} + m_4 f_3 + m_4 g_3$

* mais equações de compatibilidade:

• $a_{11} + m_1 g_1 + m_2 f_1 = 0$

• $b_{11} + a_{12} + m_2 f_1 + m_2 (g_2 + f_2) + m_3 g_2 = 0 \quad [m_4 = 0]$

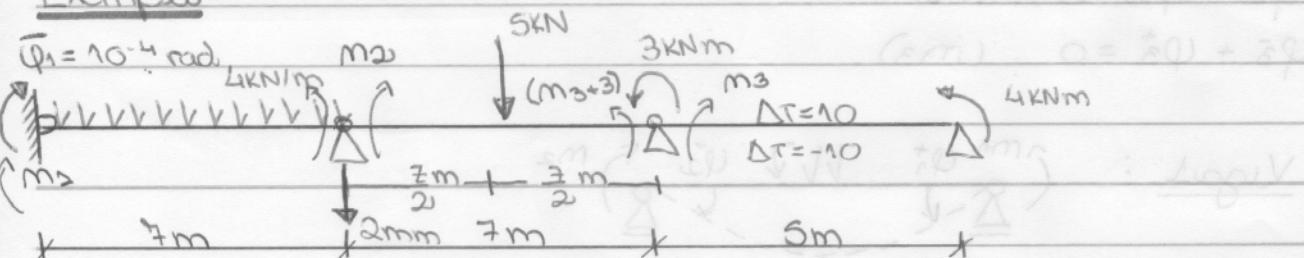
• $b_{22} + m_2 f_2 + m_3 (g_2 + f_2) + m_4 f_3 = 0$

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na forma matricial:

$$\begin{bmatrix} q_1 & g_1 & 0 \\ g_1 & g_1 + g_2 & g_2 \\ 0 & g_2 & g_2 + g_3 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -\alpha_2 \\ -(B_1 + \alpha_2) \\ -(B_2 + \alpha_3) \end{bmatrix}$$

Exemplo:



Seções de todos os vigas

0.3m	$E = 25000 \text{ MPa}$
0.2m	$\nu = 10^{-5} \text{ }^{\circ}\text{C}^{-1}$

Estrutura hiperestática \rightarrow abrir vínculos

fator de escala: $EcIc = (EI)_m$ $(EI)_m = E \cdot 0.2 \cdot (0.13)^3$

Equações de compatibilidade: $(EI)_m = 11250 \text{ KNm}^2$

$$\begin{cases} q_1^+ = EcIc \times \Delta \theta^{-4} = 1,125 \\ q_2^- + q_2^+ = 0 \\ q_3^- + q_3^+ = 0 \end{cases}$$

Caracterização da viga 1

Viga: $L' = L = 7$

$$q_1 = g_1 = \frac{4}{3}$$

$$g_2 = \frac{7}{6}$$

Devido ao carregamento (distribuído) $E = D = \frac{PL^2}{4} = 49$

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$$\alpha = \beta = 49 \cdot \frac{4}{6} = 57,17$$

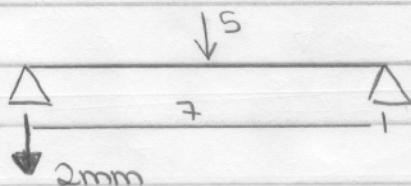
Devido ao recalque:

$$\alpha = 2 \times 10^{-3} \quad E_c I_c = 3,21 \quad \beta = -3,21$$

$$\alpha_s = 60,38$$

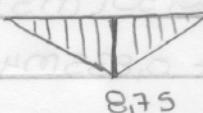
$$\beta_s = 53,96$$

- Caracterizações da viga 2:



$$\left\{ g = g' = \frac{\pi}{3} \quad g = \frac{\pi}{6} \right.$$

- * Estado de deformação



- * Estado de carregamento



$$\alpha = E_c I_c \int_{0}^{L} m(x) dx = 1 \cdot 3,5 \left(\frac{1}{2}(2,0 + 8,75) + \frac{1}{2}(0 + 2 \times 8,75) \right)$$

$$+ \frac{1}{3} \cdot 3,5 \times 0,5 \times 8,75 = 15,31$$

$$\therefore \text{Devido ao carregamento: } \alpha = 15,31 \quad \beta = 15,31$$

$$\text{Devido ao recalque: } \alpha = -3,21 \quad \beta = 3,21$$

$$\alpha_2 = 12,11$$

$$\beta_2 = 18,52$$

$$E_{24,06} = 13$$

$$P.P. = 2m$$

$$25,0 = 2m$$

$$P_{CIMA} = 12,5$$

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Caracterizações da viga 3 (semente temperatura)

$$L = L' = 5 \quad \left\{ \begin{array}{l} g = g' = \frac{5}{3} \\ \gamma = \frac{5}{6} \end{array} \right.$$

$$\frac{d\ell}{dx} = \frac{\Delta \cdot \Delta T}{l} = \frac{10^{-5} \cdot (-20)}{0.3} = -66,7 \times 10^{-5}$$

$$\alpha = E_c I_c \cdot (-66,7 \cdot 10^{-5}) \cdot \frac{5}{2} = -18,75 \quad \text{e} \beta = -18,75$$

Equações fundamentais:

$$q_1^+ = 60,38 + 2,33 m_2 + 1,167 m_3$$

$$q_2^- = 53,96 + 1,167 m_1 + 2,33 m_2$$

$$q_2^+ = 12,1 + 2,33 m_2 + 1,167 (m_3 + 3)$$

$$q_3^- = 18,52 + 1,167 m_2 + 2,33 (m_3 + 3)$$

$$q_3^+ = -18,75 + 1,167 m_3 + 0,833 m_4 \quad m_4 = 4 \text{ KNm}$$

Equações de compatibilidade:

$$60,38 + 2,33 m_1 + 1,167 m_2 = 1,125$$

$$66,06 + 1,167 m_2 + 4,006 m_3 + 1,167 m_3 + 3,5 = 0$$

$$-0,23 + 1,167 m_2 + 4 \cdot m_3 + 7 + 0,833 \cdot 4 = 0$$

$$\begin{bmatrix} 2,33 & 1,167 & 0 \\ 1,167 & 4,006 & 1,167 \\ 0 & 1,167 & 4 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -59,225 \\ -69,56 \\ -3,50 \end{bmatrix}$$

$$m_1 = -20,48$$

$$m_2 = -9,9$$

$$m_3 = 0,36$$

$$q_1^+ = 11083$$

$$q_2^- = 7,04$$

$$q_2^+ = -7,04$$

$$q_3^- = 14,79$$

$$q_3^+ = -14,99$$