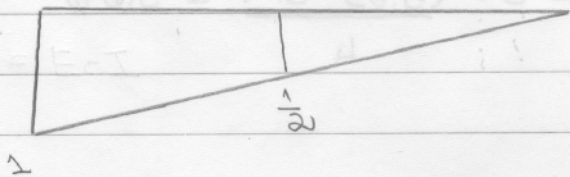


$$\text{obs: } \int_a^b \left(\frac{1-x}{8} \right)^2 dx = -\frac{1}{3} \cdot 8 \left(\frac{1-x}{8} \right)^3 \Big|_a^b$$

$$g' = 10^5 \left(-\frac{1}{3} \cdot 8 \left(\frac{1-x}{8} \right)^3 \Big|_0^4 \right) + 5 \cdot \left(-\frac{1}{3} \cdot 8 \left(\frac{1-x}{8} \right)^3 \Big|_4^8 \right)$$

$$\{g'\} = 10,40$$

Outra alternativa



$$\int m^2 dx = \frac{4}{6} \left(m_A (2m_A + m_B) + m_B (m_A + 2m_B) \right) = \frac{4}{6} \left(1 \left(2 + \frac{1}{2} \right) + \frac{1}{2} (2) \right)$$

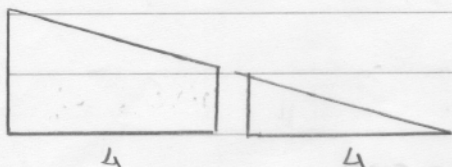
$$= \frac{7}{3}$$

$$\frac{4}{3} \left(\frac{1}{2} \right)^2 = \frac{1}{3}$$

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Continuação do exercício

$$g' = \int_0^8 \left(\frac{1-x}{8} \right)^2 \frac{1}{EI} dx ; g = \int_0^8 \left(\frac{x}{8} \right)^2 \frac{1}{EI} dx ; \gamma = \int_0^8 \frac{x}{8} \left(\frac{1-x}{8} \right) \frac{1}{EI} dx$$



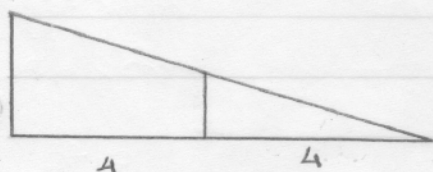
$$\int_0^4 \left(\frac{1-x}{8} \right)^2 dx = \frac{7}{3}$$

$$\int_4^8 \left(\frac{1-x}{8} \right)^2 dx = \frac{1}{3}$$

$$g' = 10^5 \cdot \frac{7}{3} + 10^5 \cdot \frac{1}{3} = 10,404$$

$$g = \frac{10^5}{26667} \cdot \frac{1}{3} + \frac{10^5}{20000} \cdot \frac{2}{3} = 7 = 12,9$$

$$\gamma = \frac{10^5}{26667} \cdot \frac{2}{3} + \frac{10^5}{20000} \cdot \frac{1}{3} = 5,83$$



$$\int_0^4 \frac{x}{8} \left(\frac{1-x}{8} \right) dx = 4 \cdot \frac{1}{6} \cdot \frac{1}{2} \left(1 + 2 \cdot \frac{1}{2} \right) = \frac{2}{3}$$

$$\int_4^8 \frac{x}{8} \left(\frac{1-x}{8} \right) dx = \frac{2}{3}$$

Cálculo de A, B:

$$A = E_c I_c \int_0^8 \left(\frac{1-x}{8} \right) \frac{du}{dx} dx = 10^5 \cdot (-10^{-3}) \cdot 4 = -400$$

$$\frac{du}{dx} = \frac{2 \cdot \Delta T}{L}$$

$$0 < x < 4 \rightarrow \frac{10^{-5} \cdot (-40)}{0,4} = -10^{-3}$$

$$4 < x < 8 \rightarrow \frac{10^{-5} \cdot (-40)}{0,4} = -10^{-3}$$

$$B = -400$$

$$gg' - \gamma^2 = (10,404)(12,9) - (5,83)^2 = 100,2$$

$$\beta_{AB} = \frac{g}{gg' - \gamma^2} = \frac{12,9}{100,2} = 0,129$$

$$\beta_{BA} = \frac{g'}{gg' - \gamma^2} = \frac{10,404}{100,2} = 0,104$$

$$M_{AB} = \frac{qg - \omega^2}{gg' - \beta^2} = \frac{-400 \cdot 12,9 + 400 \cdot 5,83}{100,2} = -28,22$$

$$M_{BA} = \frac{-\beta qg' + \omega^2}{gg' - \beta^2} = \frac{400 \times 10,4 - 400 \times 5,83}{100,2} = 18,26$$

$$\beta_{AB} \cdot \omega_{AB} = \beta_{BA} \cdot \omega_{BA} = \beta = \frac{5,83}{100,2} = 0,0582$$

$$\therefore \beta_{AB} = 0,1129$$

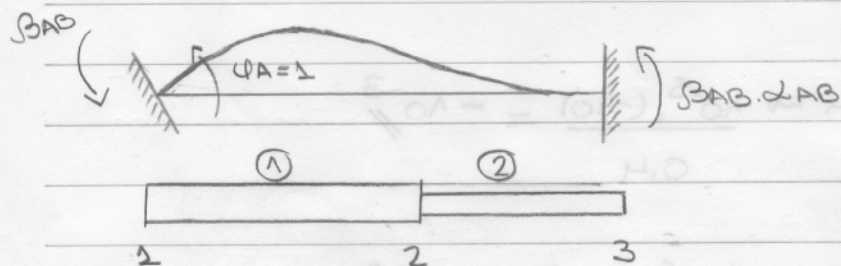
$$\beta_{BA} = 0,1104$$

$$\omega_{AB} \cdot \beta_{AB} = 0,0582$$

$$M_{AB} = -28,22$$

$$M_{BA} = 18,26$$

Resolução pelo método dos deslocamentos



Graus de Liberdade

N_2

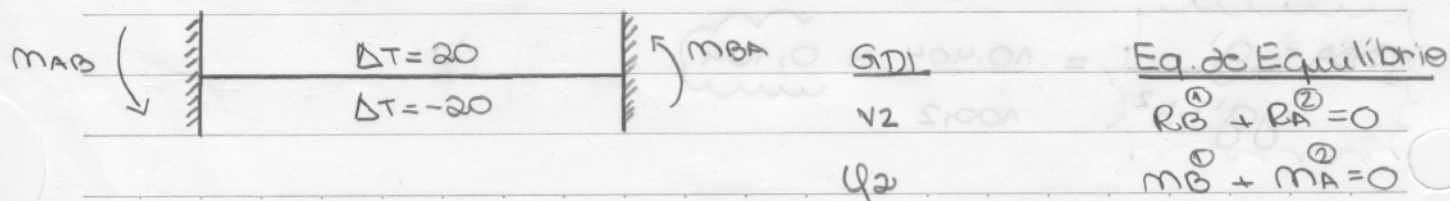
ψ_2

Eq. de Equilíbrio

$$R_B^{(1)} + R_A^{(2)} = 0$$

$$M_B^{(1)} + M_A^{(2)} = 0$$

Cálculos dos momentos de engastamento perfeito



$\Delta T = 20$

$\Delta T = -20$

GDI

ψ_2

ψ_2

Eq. de Equilíbrio

$$R_B^{(1)} + R_A^{(2)} = 0$$

$$M_B^{(1)} + M_A^{(2)} = 0$$

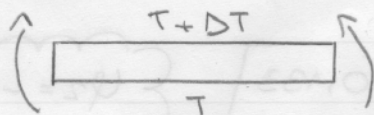
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Caracterização

Viga

$$\boxed{1} \quad L=4 \quad L' = L \frac{ECI_c}{EI} = \frac{4 \cdot 10^5}{26667} = 15 \quad \beta_{AB} = \frac{4}{15} = 0,267$$

temperatura: $M_{AB} = \frac{\Delta T}{L} \cdot EI = \frac{10^{-5} \cdot (-40)}{0,4} \cdot 26667 = -26,67$



$$M_{BA} = 26,67$$

Viga

$$\boxed{2} \quad L=4 \quad L' = \frac{4 \cdot 10^5}{20000} = 20 \quad \beta_{AB} = \frac{4}{20} = 0,2$$

$$M_{AB} = \frac{10^{-5} \cdot (-40)}{0,4} \cdot 20000 = -20 \quad M_{BA} = 20$$

Equações fundamentais

$$M_A^{(1)} = M_{AB}^{(1)} + 0,267 \psi_1 + 0,133 \psi_2 + 0,1 (v_1 - v_2)$$

$$M_B^{(1)} = M_{BA}^{(1)} + 0,133 \psi_1 + 0,267 \psi_2 + 0,1 (v_1 - v_2)$$

$$R_A^{(1)} = R_{AB}^{(1)} + 0,1 \psi_1 + 0,1 \psi_2 + 0,05 (v_1 - v_2)$$

$$R_B^{(1)} = R_{BA}^{(1)} - 0,1 \psi_1 - 0,1 \psi_2 - 0,05 (v_1 - v_2)$$

$$M_A^{(2)} = M_{AB}^{(2)} + 0,2 \psi_2 + 0,1 \psi_3 + 0,075 (v_2 - v_3)$$

$$M_B^{(2)} = M_{BA}^{(2)} + 0,1 \psi_2 + 0,2 \psi_3 + 0,075 (v_2 - v_3)$$

$$R_A^{(2)} = R_{AB}^{(2)} + 0,075 \psi_2 + 0,075 \psi_3 + 0,0375 (v_2 - v_3)$$

$$R_B^{(2)} = R_{BA}^{(2)} - 0,075 \psi_2 - 0,075 \psi_3 - 0,0375 (v_2 - v_3)$$

Cálculo de β_{AB} , α_{AB} , β_{BA}

$$\psi_1 = 1 \quad v_1 = 0 \quad \psi_3 = 0 \quad v_3 = 0 \quad M_{AB} = 0$$

VALOR ERRADO + CONFERIR!

$(-0,1 + 0,075)V_2$ deveria ser

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$$M_B^{(1)} + M_A^{(2)} = 0,367\varphi_2 - 0,15V_2 + 0,133\varphi_1 = 0$$

$$R_B^{(1)} + R_A^{(2)} = -0,1\varphi_1 - 0,1\varphi_2 + 0,05V_2 + 0,075\varphi_1 + 0,0375V_2$$

$$M_B^{(1)} + M_A^{(2)} = 0,367\varphi_2 - 0,025V_2 + 0,133 = 0$$

$$R_B^{(1)} + R_A^{(2)} = -0,025\varphi_2 + 0,0875V_2 - 0,1 = 0$$

$$\begin{bmatrix} 0,367 & -0,025 \\ -0,025 & 0,0875 \end{bmatrix} \begin{bmatrix} \varphi_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} -0,133 \\ 0,1 \end{bmatrix}$$

$$\begin{cases} \varphi_2 = -0,28 \\ V_2 = 1,06 \end{cases} \quad X$$

Correção:

$$\begin{bmatrix} 0,467 & -0,025 \\ -0,025 & 0,0875 \end{bmatrix} \begin{bmatrix} \varphi_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} -0,167 \\ 0 \end{bmatrix}$$

$$\begin{cases} \varphi_2 = -14,5 \\ V_2 = -4,14 \end{cases}$$

$$(5V - 1V) 4,0 + 5(5E) 1,0 + 1(5E) 0 + 84m = 1m$$

$$(5V - 1V) 4,0 + 5(5E) 0 + 1(5E) 1,0 + 49m = 0m$$

$$(5V - 1V) 200 + 5(0) 4,0 + 1(0) 1,0 + 849 = 849$$

$$(5V - 1V) 200 - 5(0) 4,0 - 1(0) 1,0 - 499 = 89$$

$$(5V - 5V) 250,0 + 5(0) 4,0 + 5(0) 0 + 84m = 1m$$

$$(5V - 5V) 250,0 + 5(0) 5,0 + 5(0) 4,0 + 49m = 8m$$

$$(5V - 5V) 250,0 + 5(0) 250,0 + 5(0) 250,0 + 849 = 89$$

$$(5V - 5V) 250,0 - 5(0) 250,0 - 5(0) 250,0 - 499 = 89$$

$$0 = 84m \quad 0 = 8V \quad 0 = 8V \quad 0 = 1V \quad 1 = 1V$$