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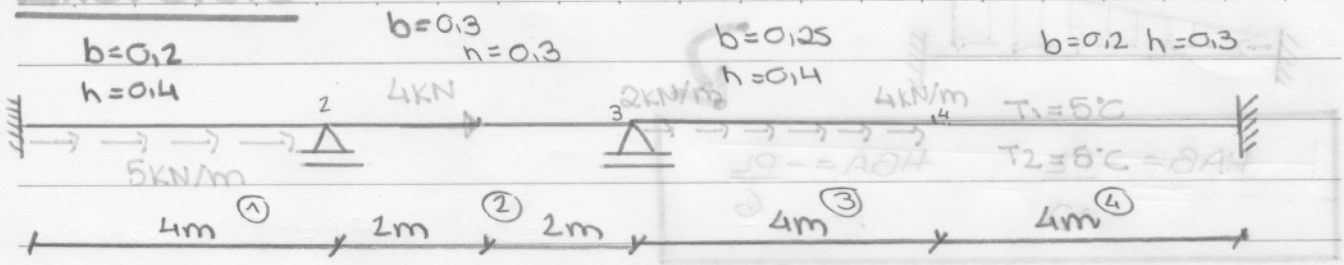
Exercício



$$H_A = H_{AB} + \frac{EA}{L E_c I_c}(y_A - u_B)$$

$$H_B = H_{BA} + \frac{EA}{L E_c I_c}(u_B - u_A)$$

$E_c I_c = (EA)_4$

$(EA)_4 = 25.000\,MPa \times 0{,}06\,m^2 = \underbrace{1{,}5 \times 10^6\,KN}$

① grau de Liberdade (GDL)

u_2

u_3

u_4

② Eq. de Eq

$H_B^{①} + H_A^{②} = 0$

$H_B^{②} + H_A^{③} = 0$

$H_B^{③} + H_A^{④} = 0$

③ caracterização das vigas

Viga 1 $L=4$ $EA = 2.10^6\,KN$

$H_{AB} = -10\,KN$ // $H_{BA} = -10\,KN$ // (fórmula)

Viga 2 $L=4m$ $EA = 2{,}25KN.10^6$

$H_{AB} = -2KN$ // $H_{BA} = -2KN$ //

Viga 3 $L=4m$ $EA = 2{,}5 \times 10^6\,KN$

$H_{AB} = -(4+4)\frac{4}{6} = -5{,}33$ // $H_{BA} = -(2+8).\frac{4}{6} = -6{,}67$ //

Viga 4 $L=4$ $EA = 1{,}5.10^6\,KN$

$H_{AB} = 75\,KN$ // $H_{BA} = -75KN$ //

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- $H_A^{(1)} = -10 + \frac{2 \cdot 10^6}{4 \cdot 1{,}5 \cdot 10^6}(u_1 - u_2) = -10 + \frac{1}{3}(u_1 - u_2)$

- $H_B^{(1)} = -10 + \frac{2 \cdot 10^6}{4 \cdot 1{,}5 \cdot 10^6}(u_2 - u_1) = -10 + \frac{1}{3}(u_2 - u_1)$

- $H_A^{(2)} = -2 + \frac{2{,}25}{4 \cdot 1{,}5}(u_2 - u_3) = -2 + 0{,}375(u_2 - u_3)$

- $H_B^{(2)} = -2 + 0{,}375(u_3 - u_2)$

- $H_A^{(3)} = -5{,}33 + 0{,}417(u_3 - u_4)$

- $H_B^{(3)} = -6{,}67 + 0{,}417(u_4 - u_3)$

- $H_A^{(4)} = 75 + 0{,}25(u_4 - u_5)$

- $H_B^{(4)} = -75 + 0{,}25(u_5 - u_4)$

$$\begin{cases} H_B^{(1)} + H_A^{(2)} = -12 + 0{,}7084\,u_2 - 0{,}375\,u_3 = 0 \\ H_B^{(2)} + H_A^{(3)} = -7{,}33 - 0{,}375\,u_2 + 0{,}792\,u_3 - 0{,}417\,u_4 = 0 \\ H_B^{(3)} + H_A^{(4)} = 68{,}33 - 0{,}417\,u_3 + 0{,}667\,u_4 = 0 \end{cases}$$

$$\begin{bmatrix} 0{,}7084 & -0{,}375 & 0 \\ -0{,}375 & +0{,}792 & -0{,}417 \\ 0 & -0{,}417 & 0{,}667 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 7{,}33 \\ -68{,}33 \end{bmatrix}$$

$u_2 = -29{,}27$

$u_3 = -87{,}27$

$u_4 = -157{,}01$

$H_A^{(1)} = -0{,}24$ $\quad H_A^{(3)} = 23{,}73$

$H_B^{(1)} = -19{,}76$ $\quad H_B^{(2)} = -35{,}73$

$H_A^{(2)} = 19{,}75$ $\quad H_A^{(4)} = 35{,}75$

$H_B^{(2)} = -23{,}75$ $\quad H_B^{(4)} = -35{,}75$

(OK!)

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Diagramas (FAZER) → FTOOL!

Exercício

Dado: $\alpha = 13$ $\quad \beta = 10$ $\quad L = 4m$

$R_A = 5$ $\quad R_B = 8$

$g' = 1{,}8$ $\quad g = 1{,}7$ $\quad g = 0{,}8$

calcular: M_{AB}, M_{BA}, R_{AB}, R_{BA}, β_{AB}, α_{AB}, β_{BA}, α_{BA}

① Equações fundamentais

$$\begin{cases} \varphi_A = -\alpha + m_A \cdot g' - m_B \cdot g \\ \varphi_B = \beta - m_A g + m_B \cdot g \end{cases} \qquad \begin{cases} \varphi_A = -13 + m_A \cdot 1{,}8 - m_B 0{,}8 \\ \varphi_B = 10 - m_A \cdot 0{,}8 + m_A 1{,}7 \end{cases}$$

$$\begin{bmatrix} \varphi_A + 13 \\ \varphi_B - 10 \end{bmatrix} = \begin{bmatrix} 1{,}8 & -0{,}8 \\ -0{,}8 & 1{,}7 \end{bmatrix} \begin{bmatrix} m_A \\ m_B \end{bmatrix}$$

$$\begin{bmatrix} 1{,}8 & -0{,}8 \\ -0{,}8 & 1{,}7 \end{bmatrix}^{-1} = \begin{bmatrix} 1{,}7 & 0{,}8 \\ 0{,}8 & 1{,}8 \end{bmatrix} \frac{1}{1{,}8 \times 1{,}7 - 0{,}8^2}$$

$$\begin{bmatrix} m_A \\ m_B \end{bmatrix} = \frac{1}{2{,}42} \begin{bmatrix} 1{,}7 & 0{,}8 \\ 0{,}8 & 1{,}8 \end{bmatrix} \begin{bmatrix} \varphi_A + 13 \\ \varphi_B - 10 \end{bmatrix}$$

β_{AB} $\qquad$ $\beta_{AB}\alpha_{AB}$

$$m_A = \frac{1{,}7}{2{,}42}\varphi_A + \frac{0{,}8}{2{,}42}\varphi_B + \frac{(1{,}7 \times 13 - 0{,}8 \times 10)}{2{,}42} \qquad M_{AB} = 5{,}83$$

$$m_B = \frac{0{,}8}{2{,}42}\varphi_A + \frac{1{,}8}{2{,}42}\varphi_B + \frac{(0{,}8 \times 13 - 1{,}8 \times 10)}{2{,}42} \qquad M_{BA} = -3{,}14$$

$\beta_{BA}\alpha_{BA}$ $\qquad$ β_{BA}

spiral

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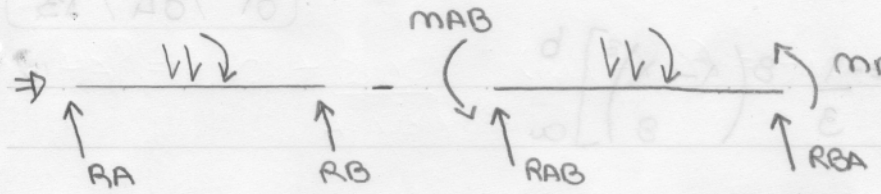
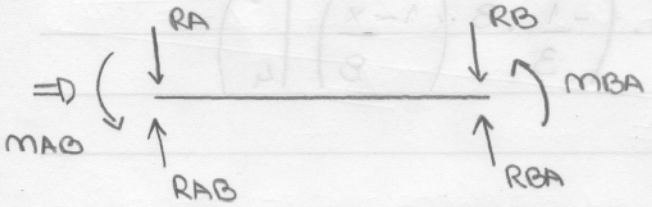


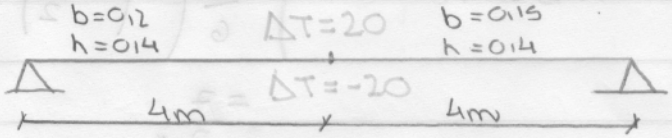


$R_{BA} \cdot L = R_B \cdot L - (M_{AB} + M_{BA})$

$R_{BA} = 8 - \frac{(5{,}83 - 3{,}14)}{4} = 7{,}33 //$

$R_{AB} = 5 + \frac{(5{,}83 - 3{,}14)}{4} = 5{,}67 //$

Exercício



Calcular β_{AB}, β_{BA}, α_{AB}, α_{BA}, M_{AB}, M_{BA} da viga completa.

• Cálculo de g' e f

m=1

$$g' = E_c I_c \int_0^8 \left(1 - \frac{x}{8}\right)^2 \cdot \frac{1}{EI(x)}\, dx$$

$0 < x < 4 \qquad EI = 25 \cdot 10^6 \cdot \frac{0{,}2 \cdot (0{,}4)^3}{12} = 26667$

$4 < x < 8 \qquad EI = 25 \cdot 10^6 \cdot \frac{0{,}15 \cdot (0{,}4)^3}{12} = 20000$

$E_c I_c \cdot 10^5$

$$g' = 10^5 \int_0^4 \frac{1}{26667} \left(1 - \frac{x}{8}\right)^2 dx + 10^5 \int_0^4 \frac{1}{20000} \left(1 - \frac{x}{8}\right)^2 dx$$

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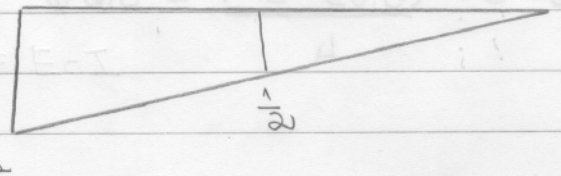
Obs: $\int_a^b \left(1-\frac{x}{8}\right)^2 dx = -\frac{1}{3}\cdot 8\left(1-\frac{x}{8}\right)^3\Big]_a^b$

$g' = \frac{10^5}{25667}\left(-\frac{1}{3}\cdot 8\left(1-\frac{x}{8}\right)^3\Big|_0^4\right) + 5\cdot\left(-\frac{1}{3}\cdot 8\left(1-\frac{x}{8}\right)^3\Big|_4^8\right)$

$g' = 10,40$

Outra asternativa



$\int m^2 \cdot dx = \frac{4}{6}\left(mA(2mA+mB) + mB(mA+2mB)\right) = \frac{4}{6}\left(1\left(2+\frac{1}{2}\right)+\frac{1}{2}(2)\right)$

$= \frac{7}{3}$ //

$\frac{4}{3}\left(\frac{1}{2}\right)^2 = \frac{1}{3}$ //

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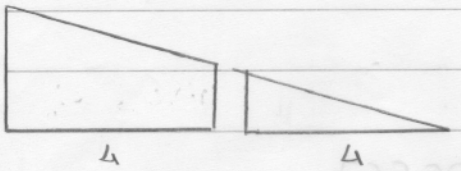
Continuação do exercício

$g' = \int_0^8 \left(1-\frac{x}{8}\right)^2 \frac{1}{EI}\,dx \quad ; \quad g = \int_0^8 \left(\frac{x}{8}\right)^2 \frac{1}{EI}\,dx \quad ; \quad y = \int_0^8 \frac{x}{8}\left(1-\frac{x}{8}\right)\frac{1}{EI}\,dx$



$\int_0^4 \left(1-\frac{x}{8}\right)^2 dx = \frac{7}{3} \qquad \int_4^8 \left(1-\frac{x}{8}\right)^2 dx = \frac{1}{3}$

$g' = \frac{10^5}{25667}\cdot\frac{7}{3} + \frac{10^5}{20000}\cdot\frac{1}{3} = 10,404$