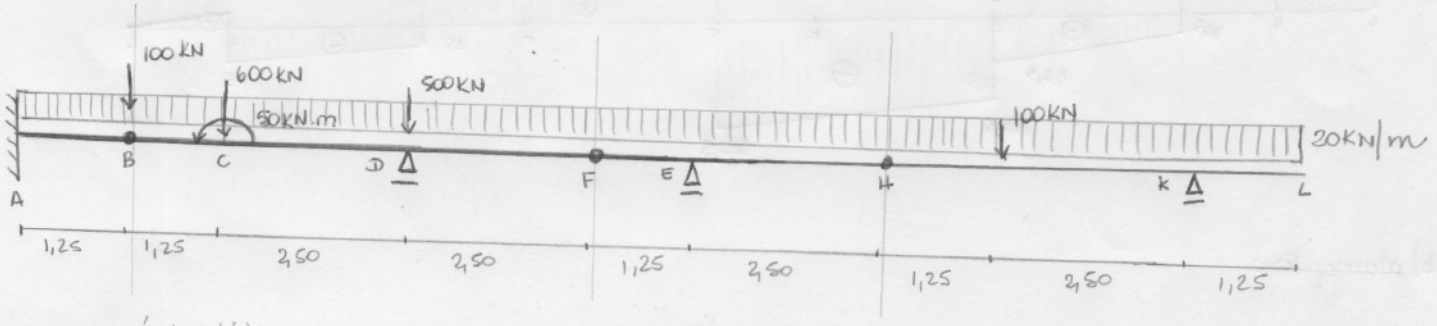
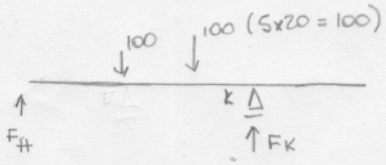
Leticia

Viga gerber

1) Determinar os diagramas de esforços solicitantes e reações de apoio devido à ação simultânea do carregamento indicado no desenho e mais um recalque vertical de 8mm para baixo no apoio D, e um efeito térmico de aumento de 20°C nas fibras superiores do trecho AL.



como a viga é isostática, o recalque e a variação na temperatura não causam esforços, somente deformação e deslocamento.

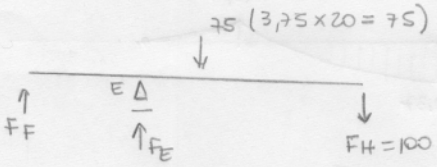


$F_H + F_K = 100 + 100 = 200$

$\Sigma M_H = 0 \rightarrow 100 \cdot 1,25 + 100 \cdot 2,5 - F_K \cdot 3,75 = 0$

$F_K = 100\ KN$

$F_H = 100\ KN$

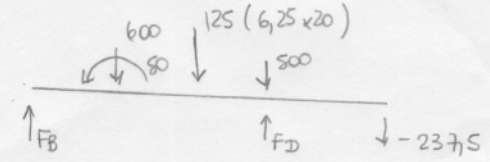


$F_F + F_E = 75 + 100 = 175$

$\Sigma M_F = 0 \rightarrow -F_E \cdot 1,25 + 75 \cdot 1,875 + 100 \cdot 3,75 = 0$

$F_E = \frac{515,625}{1,25} = 412,5\ KN \rightarrow F_E = 412,5\ KN$

$F_F = -237,5\ KN$



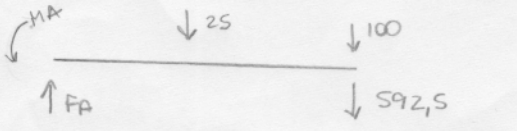
$F_B + F_D = 600 + 500 - 237,5 + 125 = 987,5$

$\Sigma M_B = 0 \rightarrow 600 \cdot 1,25 + 125 \cdot 3,125 + 500 \cdot 3,75 - 50 - 237,5 \cdot 6,25 - F_D \cdot 3,75 = 0$

$750 + 390,625 + 1875 - 50 - 1484,375 - F_D \cdot 3,75 = 0$

$F_D = 395\ KN$

$F_B = 592,5\ KN$



$125 + 592,5 = F_A$

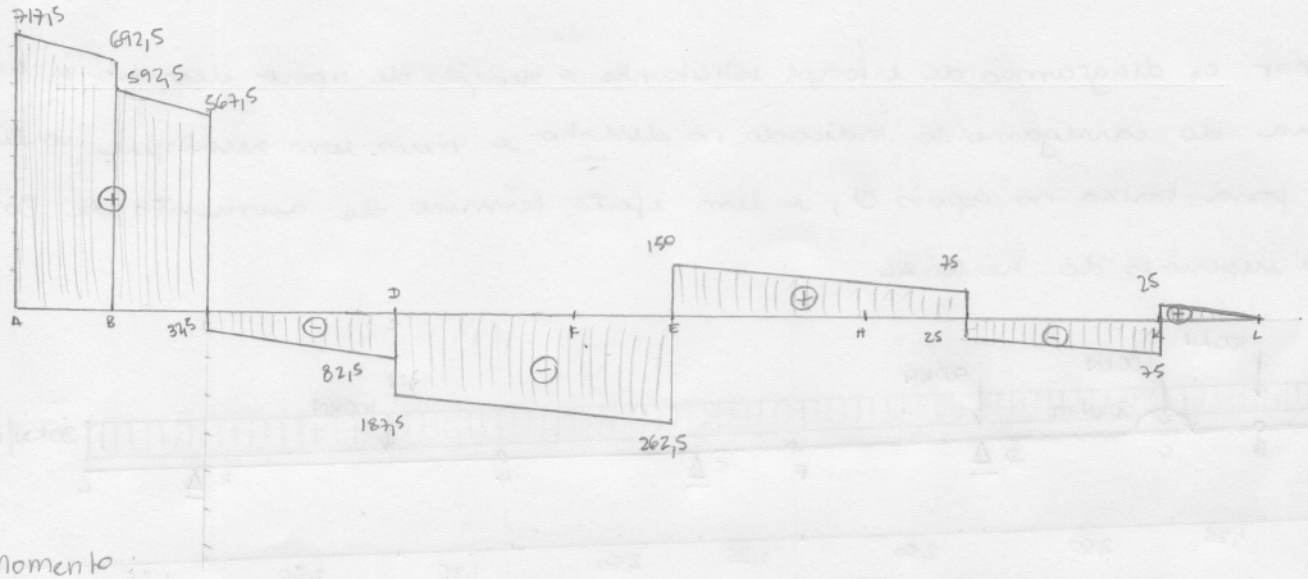
$F_A = 715,5\ KN$

$\Sigma M_A = 0 \rightarrow 25 \cdot 0,625 + 100 \cdot 1,25 + 592,5 \cdot 1,25 - M_A = 0$

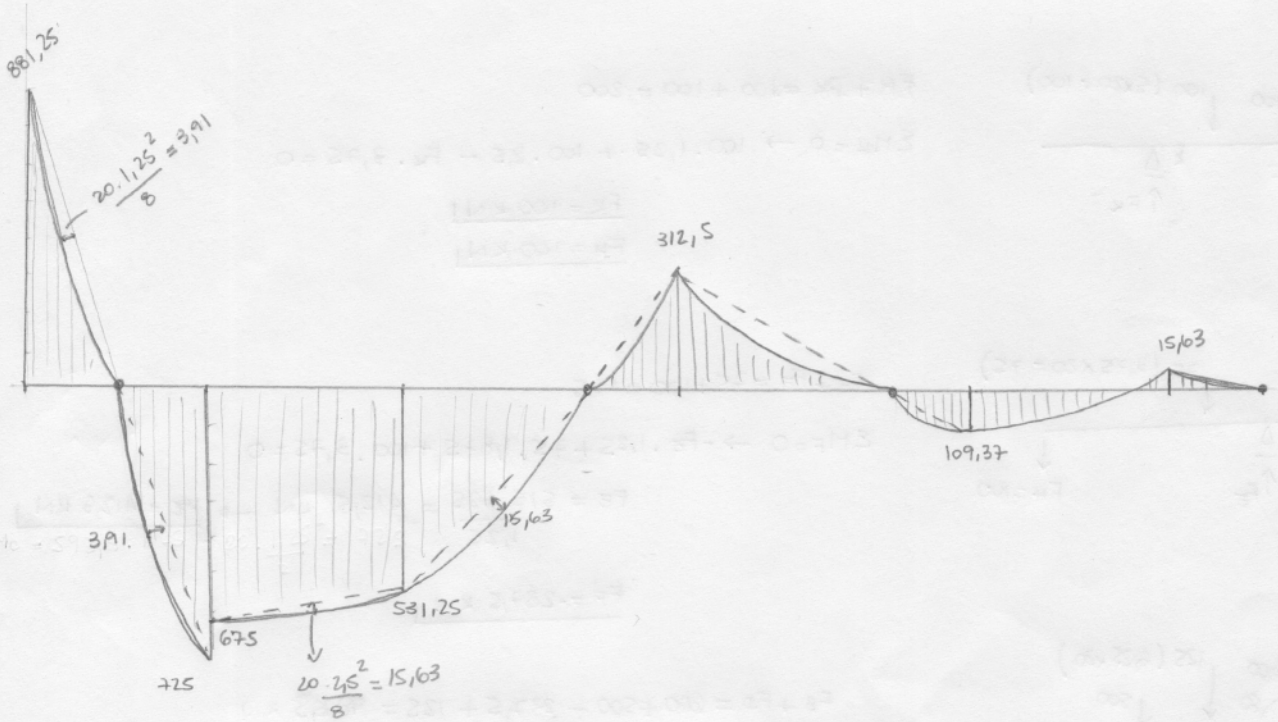
$M_A = 881,25\ KN$

Diagramas:

a) cortante:



b) Momento:



$M_C = 592,5 \cdot 1,25 - 20 \cdot \frac{1,25^2}{2} = 725$

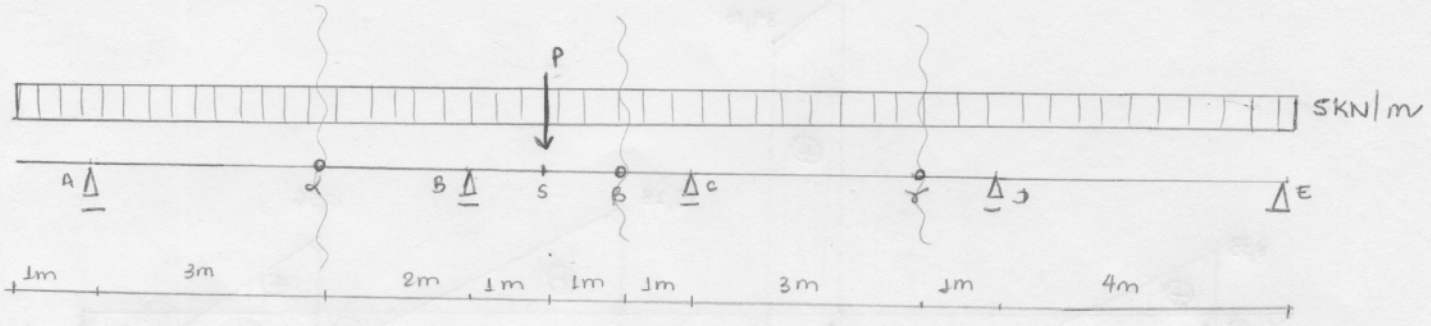
$M_D = 2375 \cdot 2,5 - 20 \cdot \frac{2,5^2}{2} = 531,25$

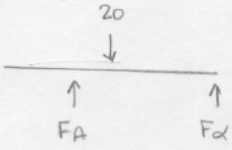
$M_E = -2375 \cdot 1,25 - 20 \cdot \frac{1,25^2}{2} = -3125$

$M_S = 100 \cdot 1,25 - 20 \cdot \frac{1,25^2}{2} = 109,37$

$M_K = -\frac{20 \cdot 1,25^2}{2} = -15,63$

2) Determinar P para que $M_B = M_C$. Traçar diagramas de cortante e momento:



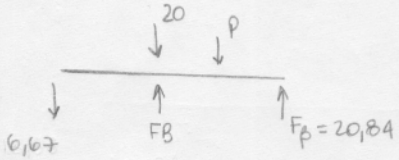


$F_A + F_\alpha = 20$

$\Sigma M_\gamma = 0 \rightarrow F_A \cdot 3 - 20 \cdot 1 = 0$

$F_A = 20/3 = 13{,}33\ KN$

$F_\alpha = 20 - 13{,}33 = 6{,}67\ KN$



$M_B = -6{,}67 \cdot 2 - 5 \cdot 2 \cdot 1 = -23{,}34\ KN\ m$

$\rightarrow -23{,}34 = M_C = -F_\beta \cdot 1 - 5 \cdot 1 \cdot 0{,}5 = -F_\beta - 2{,}5$

$F_\beta = 20{,}84\ KN$

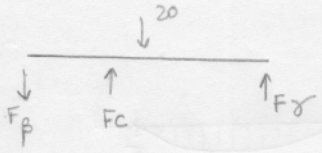
$20 + 6{,}67 + P = 20{,}84 + F_B$

$\Sigma M_\beta = 6{,}67 \cdot 4 + 20 \cdot 2 + P - F_B \cdot 2 = 0$

$66{,}68 + P = 2F_B$

$66{,}68 + F_B - 5{,}83 = 2F_B$

$F_B = 60{,}85\ KN \rightarrow P = 55{,}02\ KN$

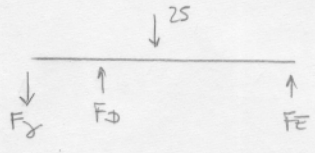


$\Sigma M_\gamma = 0 \rightarrow 20{,}84 \cdot 4 + 20 \cdot 2 - F_C \cdot 3 = 0$

$F_C = 41{,}12\ KN \rightarrow F_C + F_\gamma = F_\beta + 20$

$41{,}12 + F_\gamma = 20{,}84 + 20$

$F_\gamma = -0{,}28\ KN$



$F_\gamma + 25 = F_D + F_E$

$\Sigma M_E = 0 \rightarrow F_\gamma \cdot 5 + 25 \cdot 2{,}5 - F_D \cdot 4 = 0$

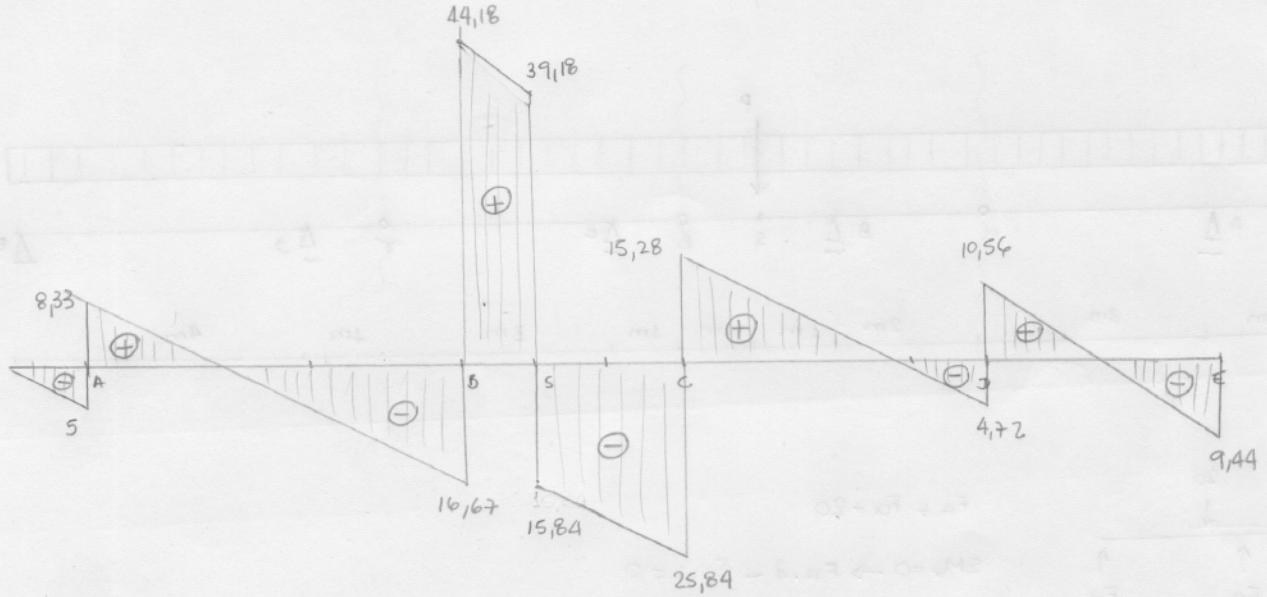
$-1{,}4 + 62{,}5 = 4F_D$

$F_D = 15{,}28 \rightarrow F_E = -0{,}28 + 25 - 15{,}28$

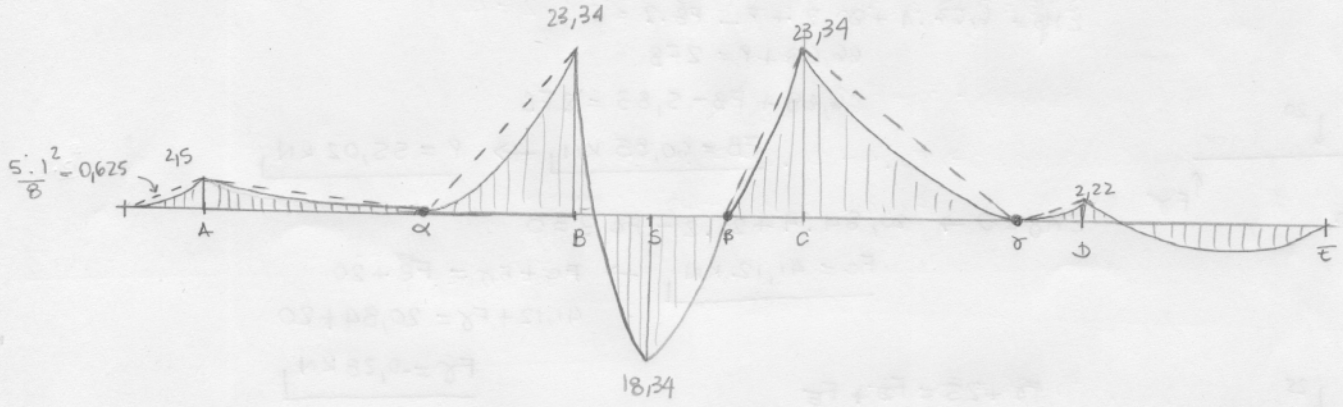
$F_E = 9{,}44\ KN$

Diagramas:

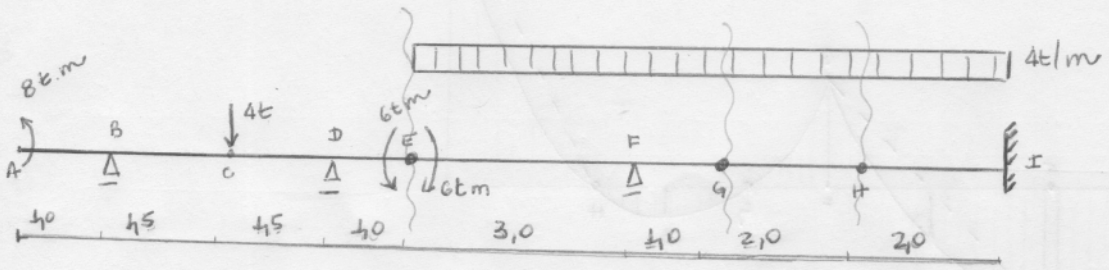
a) Cortante:

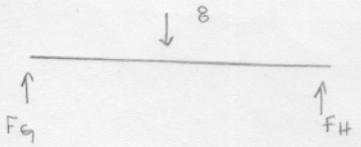


b) Momento

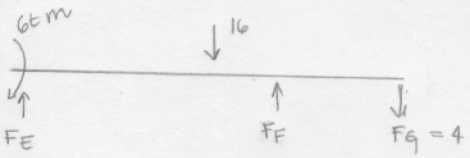


3) Traçar os diagramas de momento e cortante da viga gerber abaixo:





$F_G = F_H = 4t$

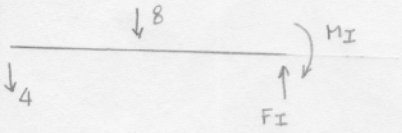


$F_E + F_F = 16 + 4 = 20t$

$\Sigma M_E = 0 \rightarrow 6 + 16 \cdot 2 + 4 \cdot 4 - F_F \cdot 3 = 0$

$F_F = 18t$

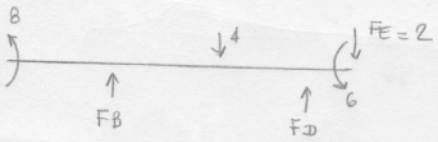
$F_E = 2t$



$F_I = 4 + 8 = 12t$

$\Sigma M_H = 0 \rightarrow 8 \cdot 1 + M_I - F_I \cdot 2 = 0$

$M_I = 16\,tm$



$F_B + F_D = 4 + 2 = 6 \rightarrow F_D = 6 - F_B$

$\Sigma M_E = 0 \rightarrow 6 - F_D \cdot 1 + 4 \cdot 2{,}5 - F_B \cdot 4 + 8 = 0$

$-F_D - 4F_B = -24 \rightarrow F_D + 4F_B = 24$

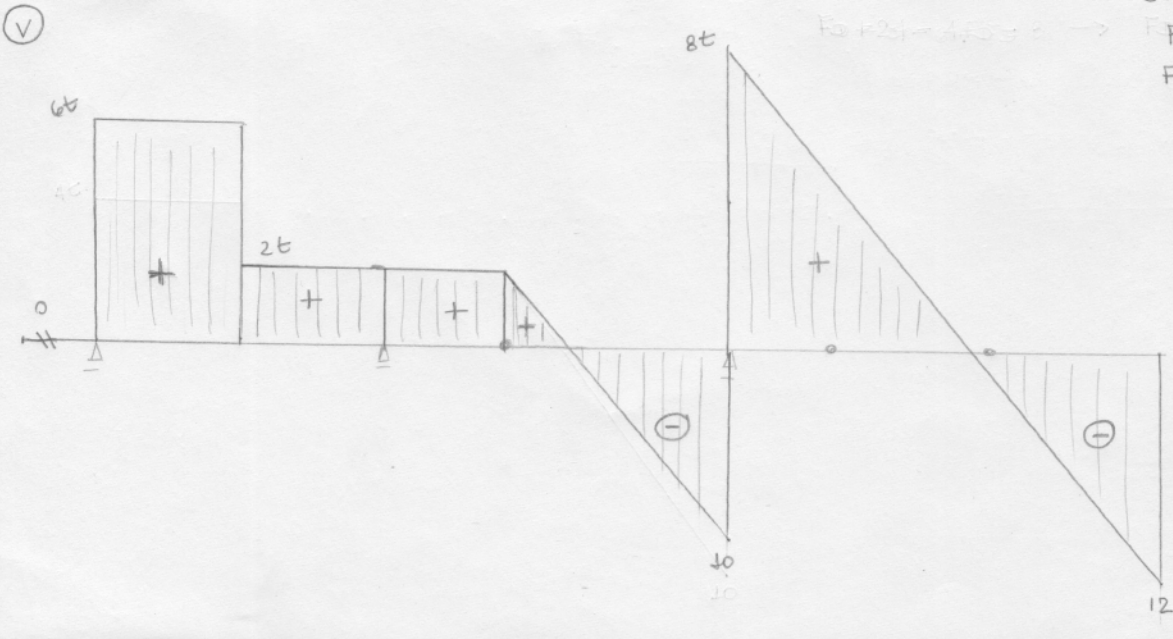
$6 - F_B + 4F_B = 24$

$3F_B = 18$

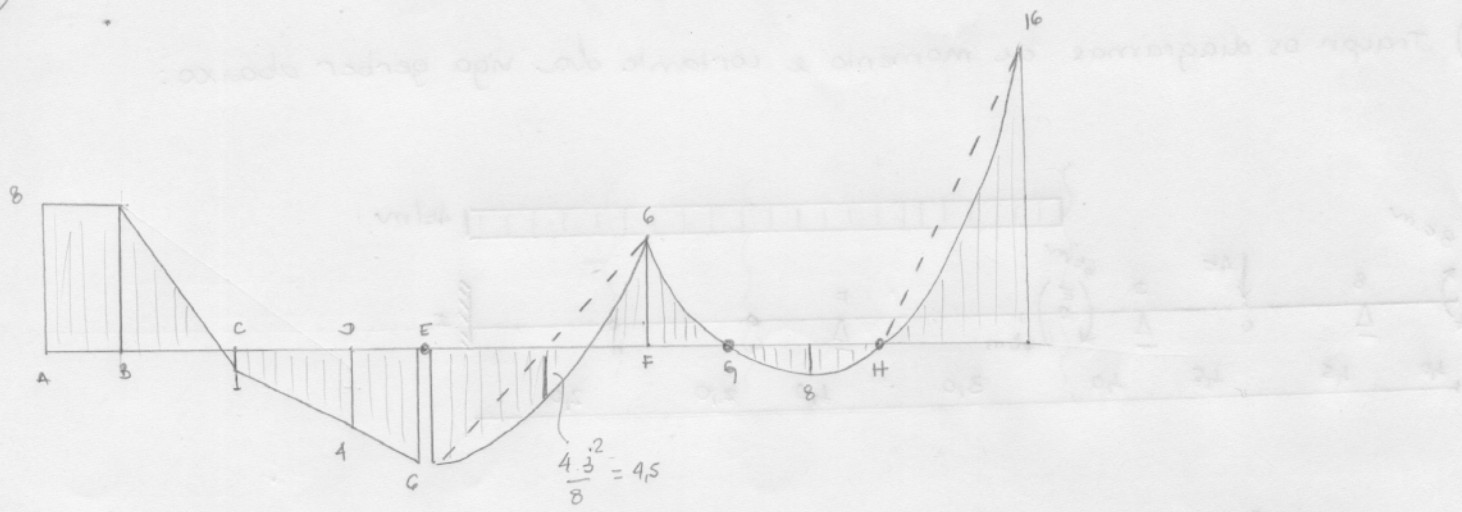
$F_B = 6t$

$F_D = 0$

(V)



(M)



$M_C = -8 + 6 \cdot 1,5 = 1$

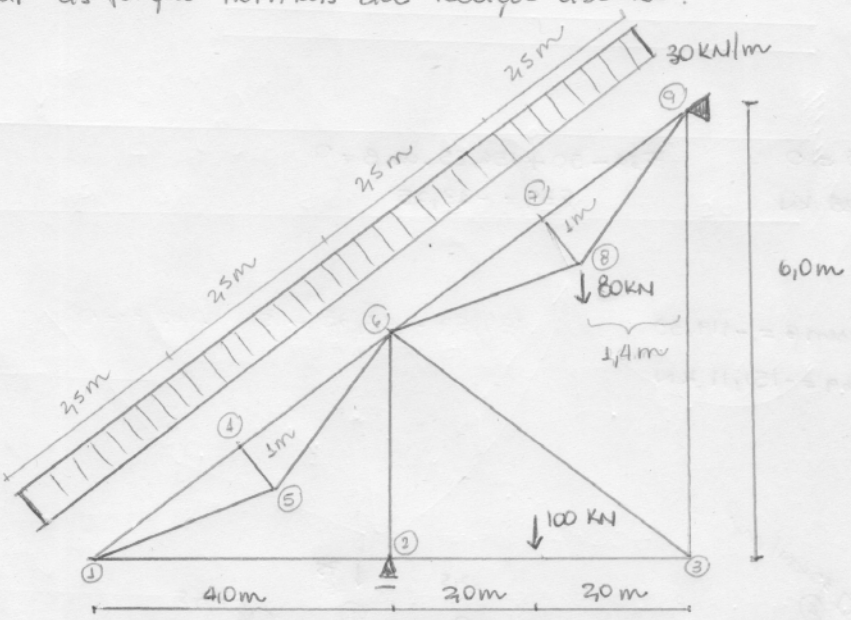
$M_D = 6 - 2 \cdot 1 = 4$

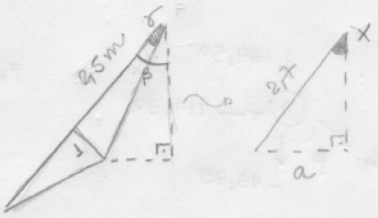
$M_F = -4 - 4 \cdot 0,5 = -6$

$M_I = -16$

Treliças

1) Determinar as forças normais da treliça abaixo:





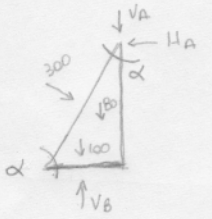
$tg\gamma = \frac{1,0}{2,5} \rightarrow \gamma = 21,80°$

$tg\beta = \frac{8}{6} \rightarrow \beta = 53,13°$

$X = 31,33°$

$\sin X = 0,52 \rightarrow \frac{a}{2,7} = 0,52$

$a = 1,4\ m$



$\sin\alpha = 0,6$

$\cos\alpha = 0,8$

$H_A = 300 . \sin\alpha = 180\ KN$

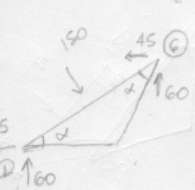
$-300.\cos\alpha - V_A - 80 - 100 + V_B = 0$

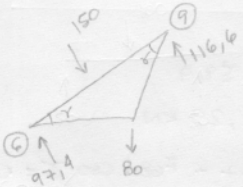
$-V_A + V_B = 420$

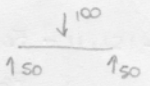
$\Sigma M_A = 0 \rightarrow 300.\sin\alpha.3 + 100.2 + 80.1,4 + 300.\cos\alpha.4,0 = V_B.4$

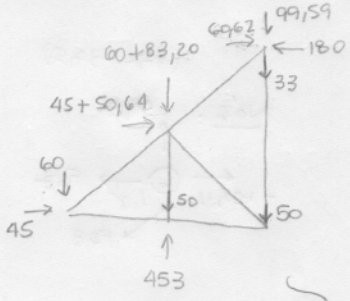
$540 + 200 + 112 + 960 = 4V_B$

$V_B = 453\ KN \rightarrow -V_A = 420 - 453 \rightarrow V_A = 33\ KN$







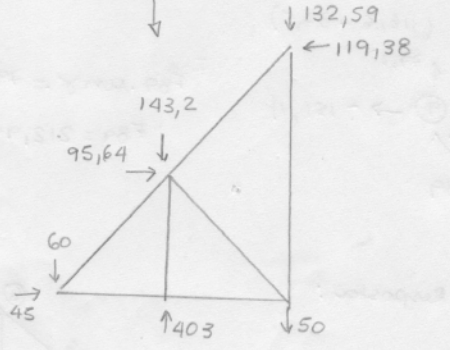


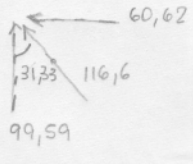
$\Sigma M_6 = 0 \rightarrow 150.2,5 + 80(4 - 1,4) = 5.F_{II}$

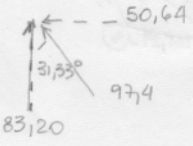
$F_{II} = 116,6\ KN$

$\Sigma M_9 = 0 \rightarrow 150.2,5 + 80.1,4 = 5\ F_I$

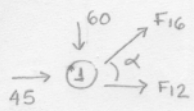
$F_I = 97,4\ KN$







Nó 1:



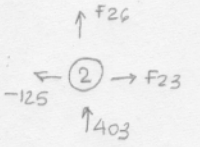
$F16 \cdot \operatorname{sen}\alpha = 60$

$F16 = 100\ kN$

$45 + F12 + 100 \cdot \cos\alpha = 0$

$F12 = -125\ kN$

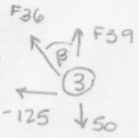
Nó 2:



$F23 = -125\ kN$

$F26 = -403\ kN$

Nó 3:



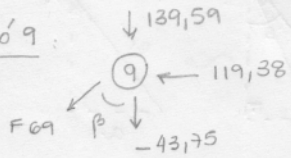
$F36 \cdot \operatorname{sen}\beta - 125 = 0$

$F36 = 156,25\ kN$

$F39 - 50 + 156,25 \cdot \cos\beta = 0$

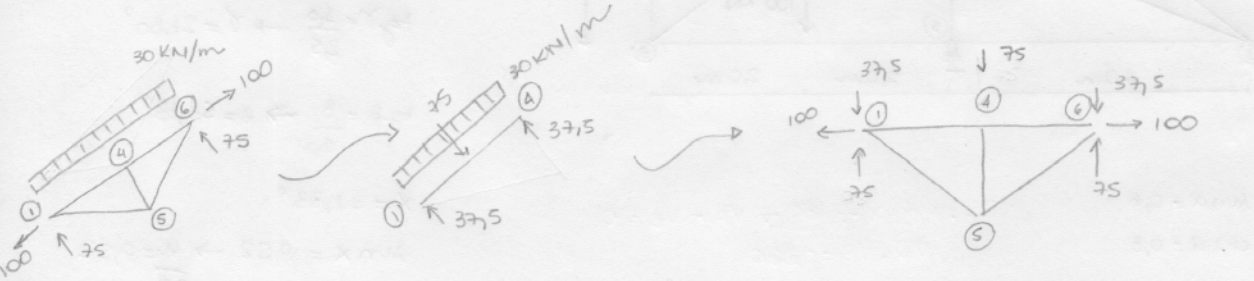
$F39 = -43,75$

Nó 9:

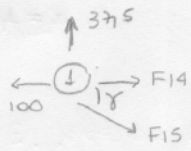


$F69 \cdot \operatorname{sen}\beta = -119,38$

$F69 = -151,11\ KN$



Nó 1:



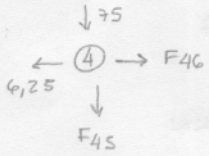
$F15 \cdot \operatorname{sen}\gamma = 37,5$

$F15 = 100,97\ KN$

$F14 + 100,97 \cdot \cos\gamma - 100 = 0$

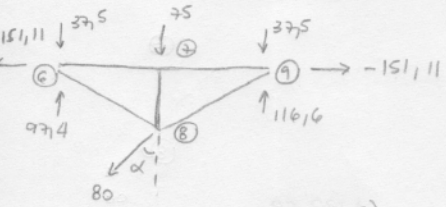
$F14 = 6,25\ kN$

Nó 4:



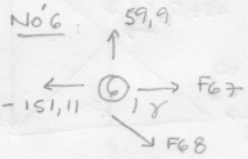
$F45 = -75\ KN$

$F46 = 6,25\ kN$



(97,4 - 37,5)

Nó 6:



$F68 \cdot \operatorname{sen}\gamma = 59,9$

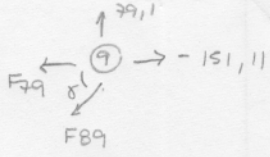
$F68 = 161,29\ KN$

$-151,11 - F67 - F68 \cdot \cos\gamma = 0$

$F67 = -300,86\ KN$

(116,6 - 37,5)

Nó 9:



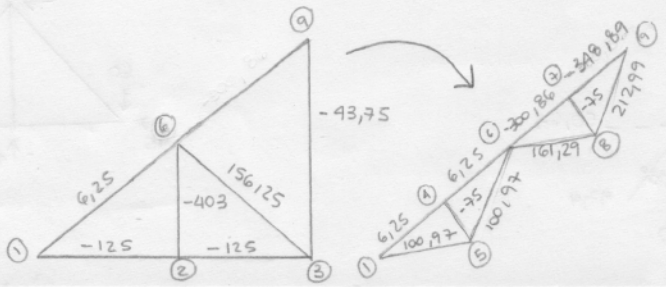
$F89 \cdot \operatorname{sen}\gamma = 79,1$

$F89 = 212,99\ KN$

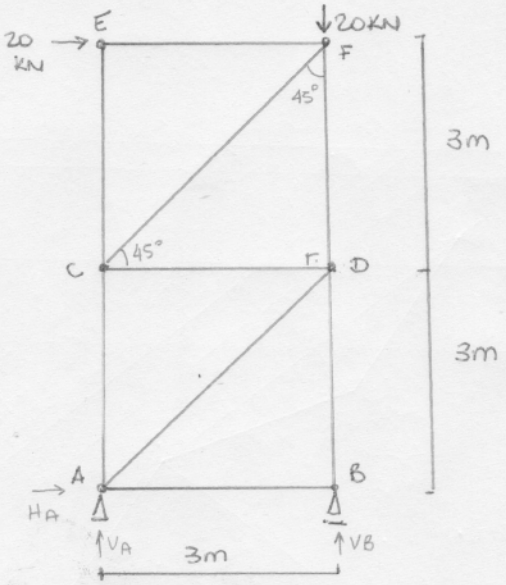
$F79 + 212,99 \cdot \cos\gamma + 151,11 = 0$

$F79 = -348,89\ KN$

Resposta:



2) determinar os esforços normais da treliça:



$\alpha \begin{cases} \sin 45 = 0{,}707 \\ \cos 45 = 0{,}707 \end{cases}$

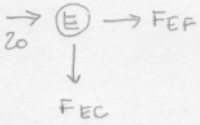
$\rightarrow V_A + V_B = 20\ kN$

$\rightarrow H_A = -20\ kN$

$\rightarrow \Sigma M_A = 0 \qquad V_B \cdot 3 - 20 \cdot 3 - 20 \cdot 6 = 0$

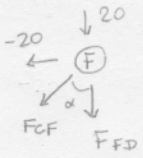
$V_B = 60\ kN$

$V_A = -40\ kN$



$F_{EC} = 0$

$F_{EF} = -20\ kN$

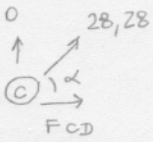


$F_{CF} \cdot \sin 45 = 20$

$F_{CF} = 28{,}28\ kN$

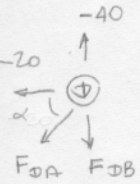
$20 + F_{FD} + F_{CF} \cdot \cos\alpha = 0$

$F_{FD} = -40\ kN$



$F_{CD} = -28{,}28 \cdot \cos\alpha$

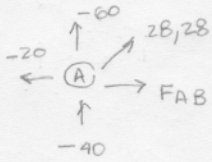
$F_{CD} = -20\ kN$



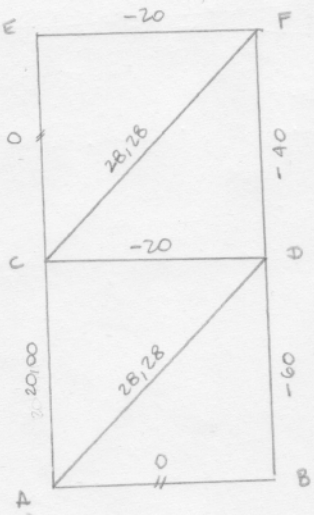
$F_{DA} = \frac{20}{\cos\alpha} = 28{,}28\ kN$

$+40 + F_{DB} + 28{,}28 \cdot \cos\alpha = 0$

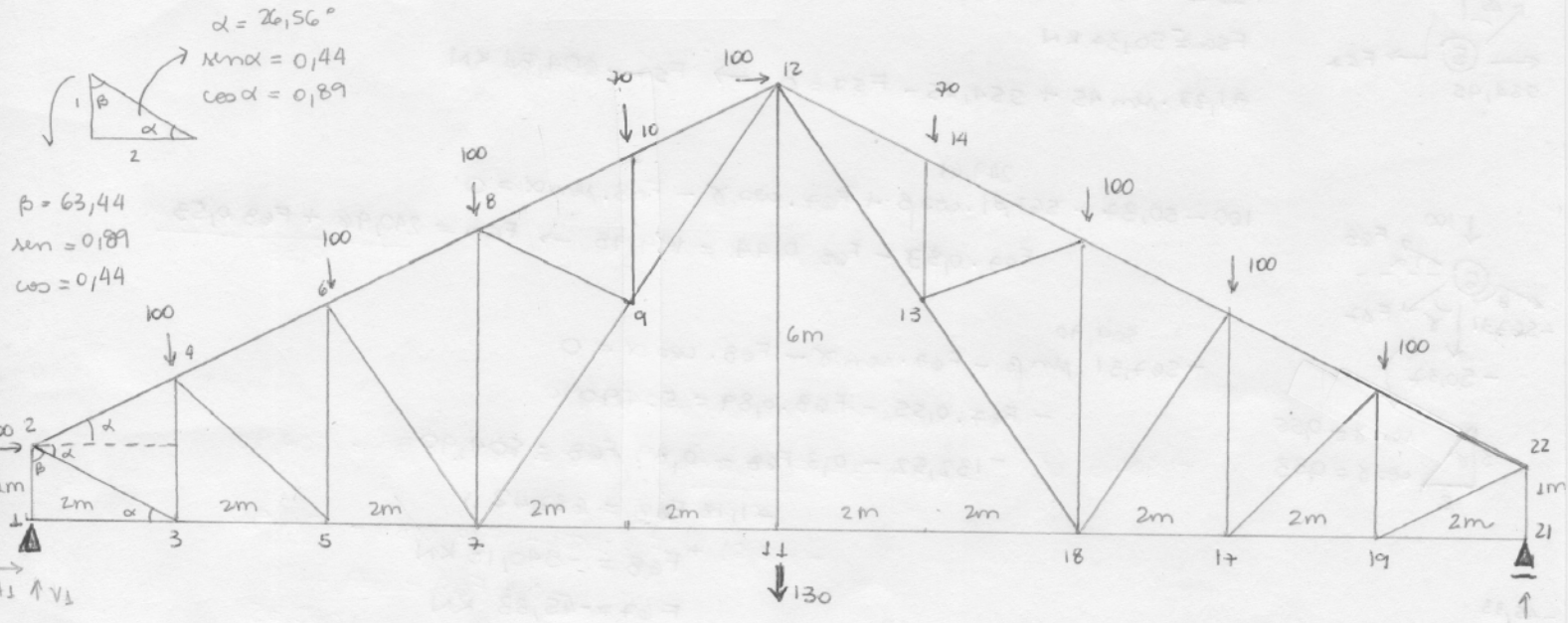
$F_{DB} = -60\ kN$



$F_{AB} = 0$



3) Seja a treliça plana mostrada na figura, com todos os nós perfeitamente articulados. As barras do banzo superior da metade esquerda têm a mesma inclinação, assim como as do banzo superior da direita. As barras 7-9 e 9-12 têm a mesma inclinação; o mesmo acontece com as barras 12-13 e 13-15. As barras têm a mesma seção transversal e são constituídas de mesmo material. Calcule o valor dos esforços solicitantes em todas as barras.



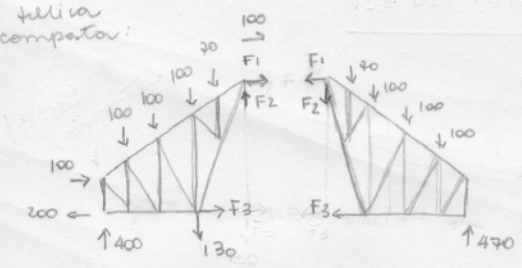
$H_1 = -100 - 100 = -200\ KN$

$V_1 + V_{21} = 740 + 130 = 870$

$\Sigma M_1 = 0 \rightarrow 100 \cdot 2 + 100 \cdot 4 + 100 \cdot 6 + 70 \cdot 8 + 70 \cdot 12 + 100 \cdot 14 + 100 \cdot 16 + 100 \cdot 18 + 130 \cdot 10 - V_{21} \cdot 20 + 100 + 600 = 0$

$V_{21} = \frac{9400}{20} = 470\ KN \rightarrow V_1 = 870 - 470 = 400\ KN$

Treliça composta:



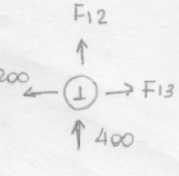
$\Sigma M_{12} = 0 \rightarrow F_3 \cdot 6 + 70 \cdot 2 + 100 \cdot 4 + 130 \cdot 4 + 100 \cdot 6 + 100 \cdot 8 + 100 \cdot 5 =$

$= 200 \cdot 6 + 400 \cdot 10$

$6F_3 = 5200 - 2960 \Rightarrow \underline{F_3 = 373,33\ KN}$

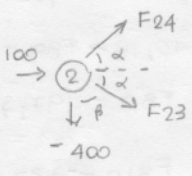
$200 - 100 - 373,33 - F_1 - 100 = 0 \rightarrow \underline{F_1 = -373,33\ KN}$

$300 + 70 - 400 + 130 - F_2 = 0 \rightarrow \underline{F_2 = 100\ KN}$



$\underline{F_{12} = -400\ KN}$

$\underline{F_{13} = 200\ KN}$



$+400 + F_{24} \cdot \sin\alpha + F_{23} \cdot \sin\alpha = 0$

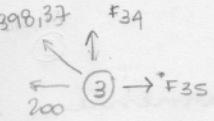
$\sin\alpha\,(+F_{24} + F_{23}) = -400$

$-F_{23} + F_{24} = -909,09 \rightsquigarrow \underline{F_{23} = +398,37\ KN}$

$100 + F_{24}\cos\alpha + F_{23}\cos\alpha = 0$

$F_{24} + F_{23} = -112,35$

$\rightarrow F_{24} + 909,09 + F_{24} = -112,35 \rightsquigarrow \underline{F_{24} = -510,72\ KN}$

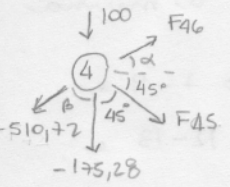


$F_{34} = 398 \cos\beta$

$F_{34} = -175,28\ KN$

$-200 - 398,37 \cdot \cos\alpha + F_{35} = 0$

$F_{35} = 554,45\ KN$



$100 - 175,28 - 510,72 \cdot \cos\beta + F_{45} \cdot \cos 45 - F_{46} \cdot \operatorname{sen}\alpha = 0$ (224,71)

$F_{45} \cos 45 - F_{46} \operatorname{sen}\alpha = 299,99 \rightarrow F_{45} \cos 45 = 299,99 + F_{46} \operatorname{sen}\alpha$

$-510,72 \cdot \operatorname{sen}\beta - F_{45} \cdot \cos 45 - F_{46} \cdot \cos\alpha = 0$

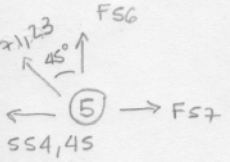
$-F_{45} \cos 45 - F_{46} \cdot \cos\alpha = 454,54$

$-299,99 - F_{46} \operatorname{sen}\alpha - F_{46} \cos\alpha = 454,54$

$-1,33\ F_{46} = 754,53$

$F_{46} = -567,31\ KN$

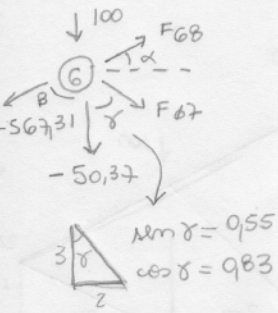
$F_{45} = 71,23\ KN$



$F_{56} = -71,23 \cdot \cos 45$

$F_{56} = -50,37\ KN$

$-71,23 \cdot \operatorname{sen} 45 + 554,45 - F_{57} = 0 \rightarrow F_{57} = 604,81\ KN$



$100 - 50,37 - 567,31 \cdot \cos\beta + F_{67} \cdot \cos\gamma - F_{68} \cdot \operatorname{sen}\alpha = 0$ (249,61)

$F_{67} \cdot 0,83 - F_{68}\ 0,44 = 199,98 \rightarrow F_{67} = 240,96 + F_{68}\ 0,53$

$-567,31 \cdot \operatorname{sen}\beta - F_{67} \cdot \operatorname{sen}\gamma - F_{68} \cdot \cos\alpha = 0$ (504,90)

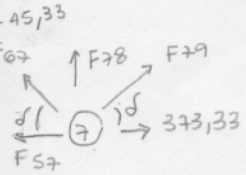
$-F_{67} \cdot 0,55 - F_{68} \cdot 0,89 = 504,90$

$-132,52 - 0,3\ F_{68} - 0,89\ F_{68} = 504,90$

$-1,18\ F_{68} = 637,42$

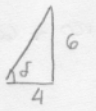
$F_{68} = -540,18\ KN$

$F_{67} = -45,33\ KN$



$-45,33 \cdot \operatorname{sen}\delta + F_{78} + F_{79} \cdot \operatorname{sen}\delta = 0$ (37,62; 0,83)

$F_{78} + F_{79} \cdot 0,83 = 37,62$

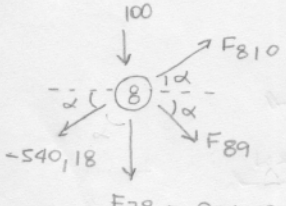


$604,81 - 45,33 \cdot \cos\delta - F_{79} \cdot \cos\delta - 373,33 = 0$ (-24,93; 0,55)

$-F_{79} \cdot 0,55 = -206,55$

$F_{79} = 375,54\ KN$

$F_{78} = -274,08\ KN$



$-540,18 \cdot \cos\alpha - F_{810} \cdot \cos\alpha - F_{89} \cdot \cos\alpha = 0$

$\cos\alpha\ (-F_{810} - F_{89}) = 480,76$

$F_{810} + F_{89} = -540,18 \rightarrow F_{810} = -540,18 - F_{89}$

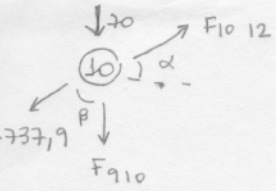
$-540,18 \cdot \operatorname{sen}\alpha - 274,08 + F_{89} \cdot \operatorname{sen}\alpha - F_{810} \cdot \operatorname{sen}\alpha + 100 = 0$ (-237,6)

$F_{89} - F_{810} = 935,63$

$F_{89} + 540,18 + F_{89} = 935,63$

$F_{89} = 197,72\ KN$

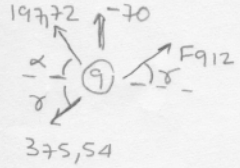
$F_{810} = -737,9\ KN$



$F1012 \cdot \cos\alpha = -737,9 \cdot \operatorname{sen}\beta$

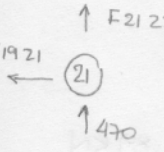
$F1012 = -737,9 \text{ KN}$

$F910 = -70 \text{ KN}$



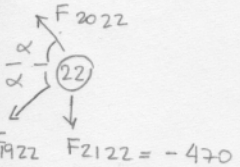
$19772 \cdot \operatorname{sen}\alpha - 70 + F912 \operatorname{sen}\gamma - 375,54 \cdot \operatorname{sen}\gamma = 0$

$F912 = 336,92 \text{ KN}$



$F2122 = -470 \text{ KN}$

$F1921 = 0$



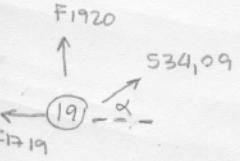
$F2022 \cdot \operatorname{sen}\alpha = F1922 \cdot \operatorname{sen}\alpha + 470 = 0$

$F2022 \cdot \cos\alpha + F1922 \cdot \cos\alpha = 0$

$F2022 = -F1922$

$F2022 = -534,09 \text{ KN}$

$F1922 = 534,09 \text{ KN}$

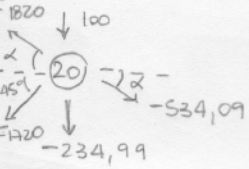


$534,09 \cdot \cos\alpha = F1719$

$F1719 = 475,34 \text{ KN}$

$534,09 \cdot \operatorname{sen}\alpha = -F1920$

$F1920 = -234,99 \text{ KN}$



$F1820 \cdot \operatorname{sen}\alpha - 100 - F1720 \cdot \operatorname{sen}45 + 234,99 + 534,09 \operatorname{sen}\alpha = 0$

$F1820 \cdot 0,44 - F1720 \cdot 0,70 = -369,99 \rightsquigarrow -F1720 \cdot 0,7 = -369,99 - F1820 \cdot 0,44$

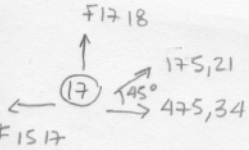
$F1820 \cdot \cos\alpha + F1720 \cdot \cos 45 = -534,09 \cdot \cos\alpha$

$F1820 \cdot 0,89 + F1720 \cdot 0,70 = -377,65$

$F1820\ 0,89 + F1820\ 0,44 + 369,99 = -377,65$

$-747,64 = F1820 \cdot 1,33 \rightarrow F1820 = -562,13 \text{ KN}$

$F1720 = 175,21 \text{ KN}$

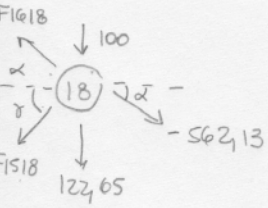


$F1718 = 175,21 \cdot 0,7$

$F1718 = 122,65 \text{ KN}$

$475,34 + 175,21 \cdot \cos 45 = F1517$

$F1517 = 597,98 \text{ KN}$



$F1618 \operatorname{sen}\alpha - 100 - F1518 \cdot \operatorname{sen}\gamma - 122,65 + 562,13 \cdot \operatorname{sen}\alpha = 0$

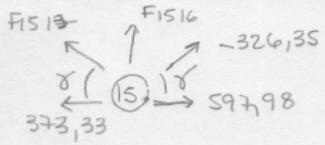
$F1618 \cdot 0,44 - F1518 \cdot 0,55 = -24,68 \rightarrow F1618 = \dfrac{-24,68 + F1518 \cdot 0,55}{0,44}$

$F1618 \cos\alpha + F1518 \cdot \cos\gamma = -562,13 \cdot \cos\alpha$ (500,29)

$-49,92 + 0,55\ F1518 + F1518 \cdot 0,83 = -500,29$

$F1518 = -326,35 \text{ KN}$

$F1618 = -464,03 \text{ KN}$



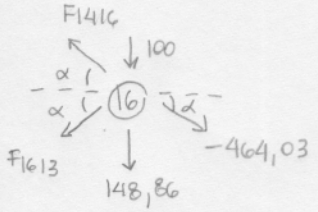
$F_{1513} \cdot \operatorname{sen}\gamma + F_{1516} - 326{,}35 \operatorname{sen}\gamma = 0$

$F_{1513} \cdot 0{,}55 + F_{1516} = 179{,}49$

$F_{1513} \cos\gamma + 373{,}33 + 326{,}35 \cos\gamma - 597{,}98 = 0$

$F_{1513} = 55{,}68 \text{ KN}$

$F_{1516} = 148{,}86 \text{ KN}$



$F_{1416} \cos\alpha + F_{1613} \cos\alpha = -464{,}03 \cos\alpha$

$F_{1416} + F_{1613} = -464{,}03$

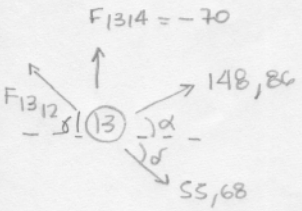
$F_{1416} \cdot \operatorname{sen}\alpha - 100 - F_{1613} \cdot \operatorname{sen}\gamma - 148{,}86 + 464{,}03 \cdot \operatorname{sen}\alpha = 0$

$(-464{,}03 - F_{1613}) \cdot 0{,}44 - F_{1613} \cdot 0{,}55 = -55{,}31$

$-0{,}44\, F_{1613} - 0{,}55\, F_{1613} = 148{,}86$

$F_{1613} = 148{,}86 \text{ KN}$

$F_{1416} = -612{,}89 \text{ KN} \rightarrow F_{1412} = -612{,}89 \text{ KN}$



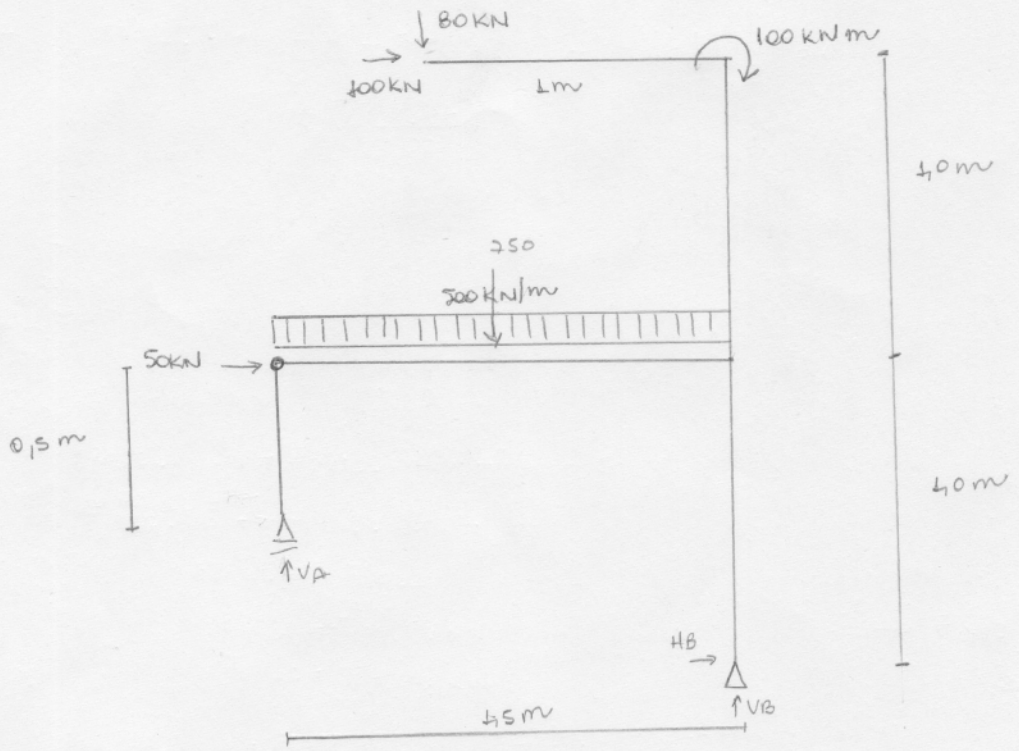
$F_{1314} + F_{1312} \cdot \operatorname{sen}\gamma + 148{,}86 \cdot \operatorname{sen}\alpha - 55{,}68 \cdot \operatorname{sen}\delta = 0$ (983)

$F_{1314} + F_{1312} \cdot 0{,}55 = -19{,}28$

$F_{1312} = 92{,}21 \text{ KN}$

Pórticos:

1) Traçar os diagramas de cortante e momento da estrutura isostática abaixo:



$\Sigma FH = 0 \longrightarrow 50 + 100 + HB = 0$

$HB = -150\ KN$

$\Sigma FV = 0 \longrightarrow 80 + 750 = VA + VB$

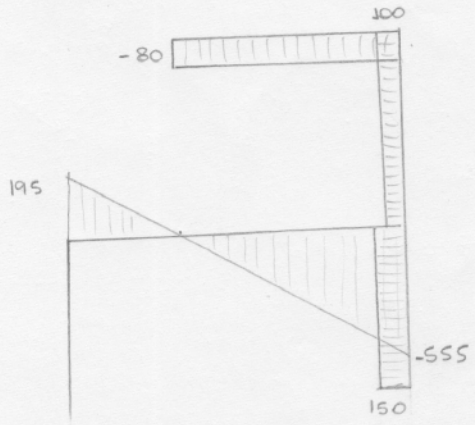
$VA + VB = 830$

$\Sigma MB = 0 \longrightarrow VA\ 1,5 + 50 \cdot 4,0 - 750 \cdot 0,75 - 80 \cdot 1 + 100 \cdot 2 + 100 = 0$

$VA = \frac{292,5}{1,5} \longrightarrow VA = 195\ KN$

$VB = 635\ KN$

(V)

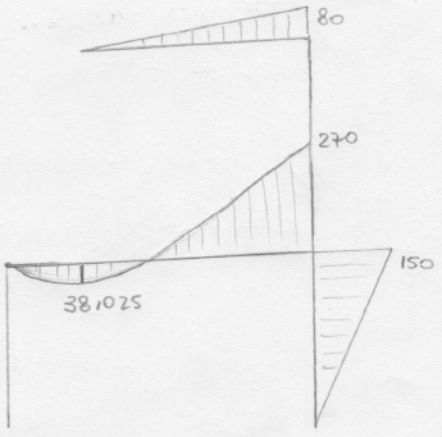


1,m — 500 KN

x — 195

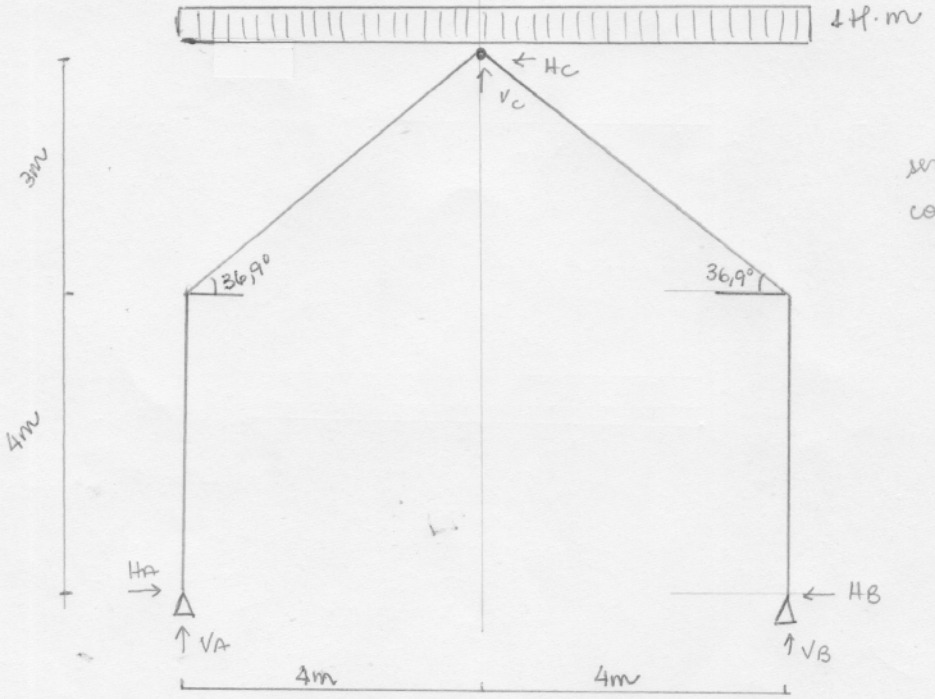
x = 0,39

(M)



$195 \times \frac{0,39}{2} = 38,025$

2) Calcular os esforços solicitantes do pórtico a seguir:



$\text{sen } 36,9 = 0,60$

$\cos 36,9 = 0,79$

$\Sigma F_H = 0 \rightarrow H_A = H_B$

$\Sigma F_V = 0 \rightarrow V_A + V_B = 8$

$\Sigma M_A = 0 \rightarrow V_B \cdot 8 - 8 \cdot 4 = 0$

$V_B = 4\,tf$

$V_A = 4\,tf$

$H_A = H_C$

$V_A + V_C = 4 \rightarrow V_C = 0$

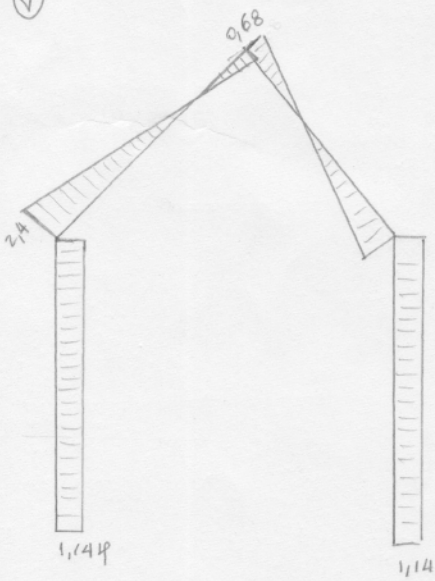
$+V_A \cdot 4 + H_A \cdot 7 + 4 \cdot 2 = 0 \rightarrow H_A 7 = 8$

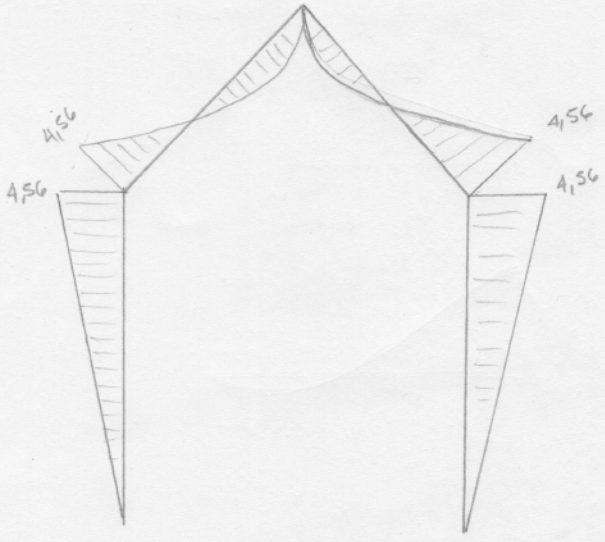
$H_A = 1,14\,tf$

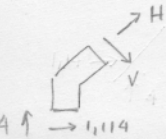
$H_C = 1,14\,tf$

$H_B = 1,14\,tf$

(V)



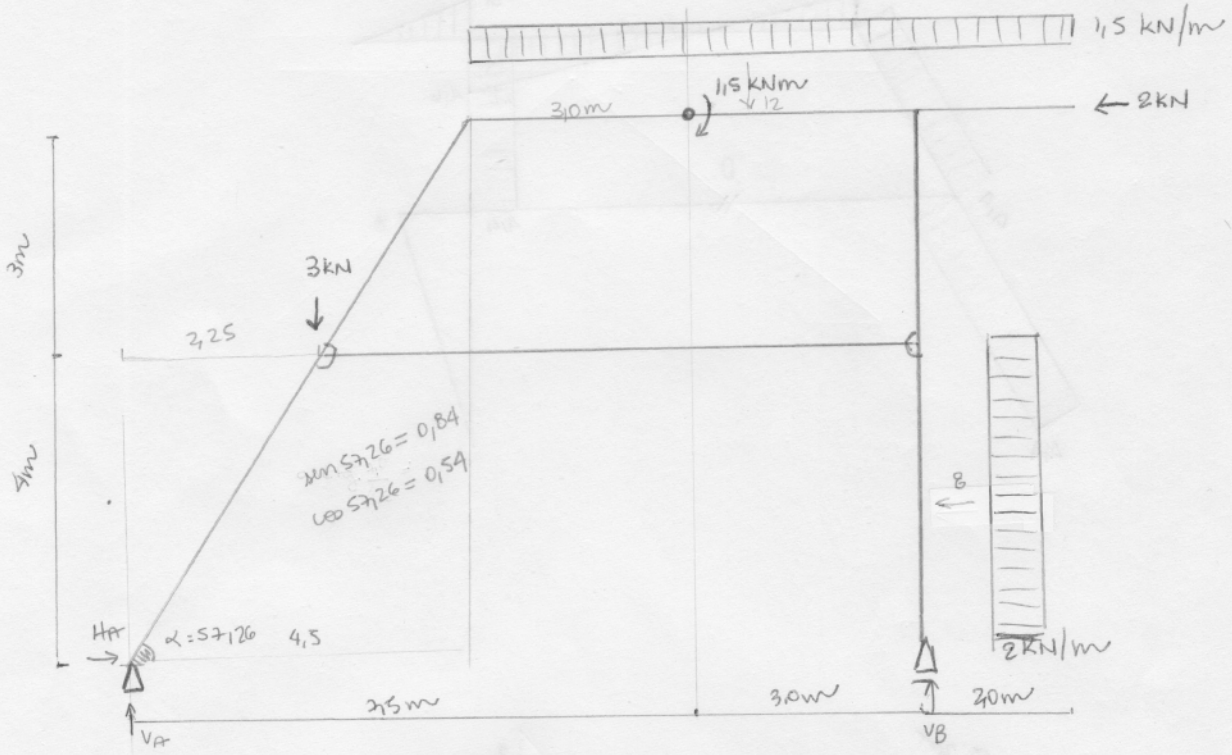




$V = -1,14 \cdot 0,6 + 4 \cdot 0,79 = 2,47\,tf$

$N = 1,14 \cdot 0,79 + 4 \cdot 0,6 = 3,30\,tf$

3) Traçar os diagramas de cortante e momento:



$\Sigma FH = 0 \rightarrow HA - 2 - 8 = 0$

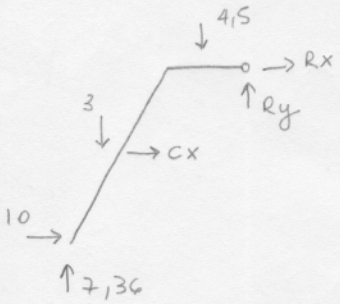
$HA = 10\ kN$

$\Sigma FV = 0 \rightarrow VA + VB = 3 + 12 = 15$

$\Sigma MA = 0 \rightarrow 3 \cdot 2,25 + 4,5 + 12 \cdot 8,5 - 2 \cdot 7 - 8 \cdot 2 - VB \cdot 10,5 = 0$

$VB = \frac{80,25}{10,5} = 7,64 \rightarrow VB = 7,64\ kN$

$VA = 7,36\ kN$



$10 + Cx + Rx = 0$

$7,36 - 3 - 4,5 + Ry = 0 \rightarrow Ry = +0,14\ kN$

$4,5 \cdot 1,5 + 3 \cdot 5,25 + 10 \cdot 7 - 7,36 \cdot 7,5 + Cx \cdot 3 = 0$

$Cx = -\frac{37,3}{3} = -12,43\ kN$

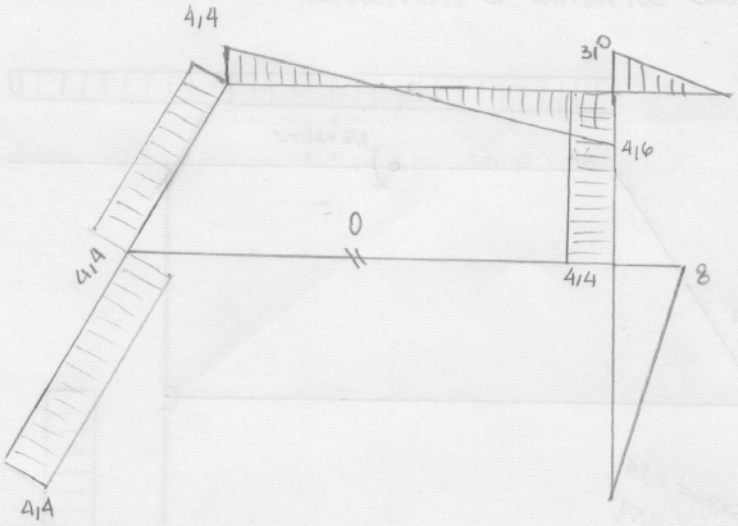
$Rx = +2,43\ kN$

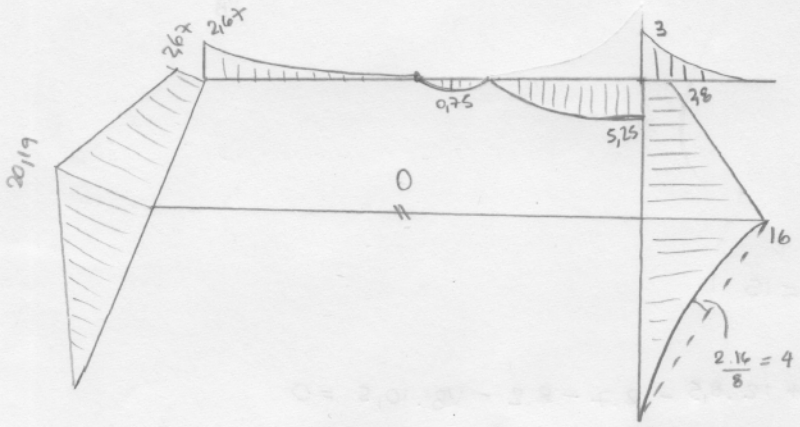
$V = 10 \operatorname{sen}\alpha - 7,36 \cdot \cos\alpha = 4,42\ kN$

$N = 10 \cos\alpha + 6,4 \operatorname{sen}\alpha = 10,77\ kN$

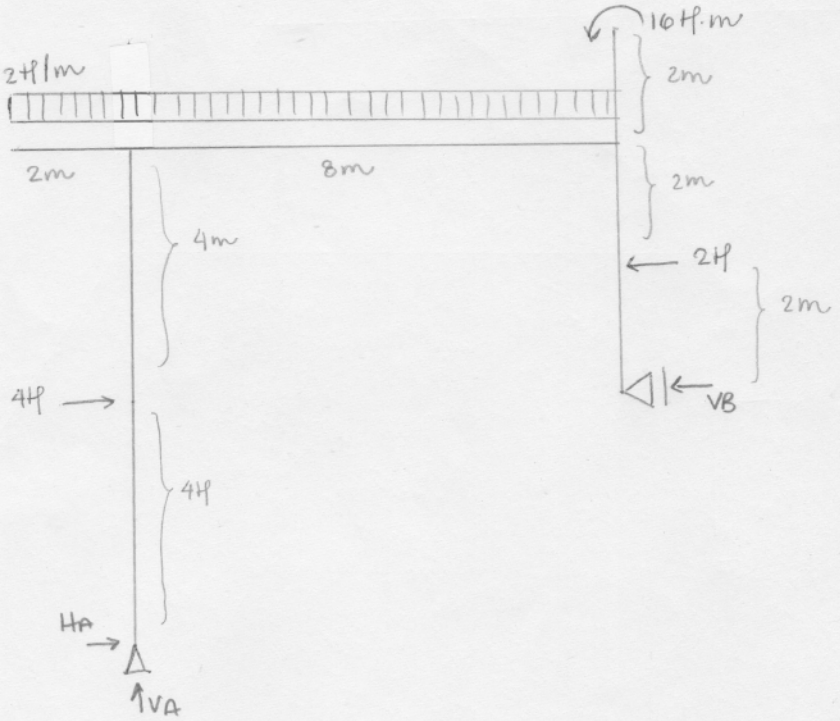
$-1,62 - 10,44 = -8,82 + 4,42 = -4,4$

(V)





4) Calcule as reações de apoio e trace os diagramas de momento e cortante



$\Sigma F_H = 0 \rightarrow 4 + H_A - 2 - V_B = 0$

$H_A - V_B = -2$

$\Sigma F_V = 0 \rightarrow V_A - 20 = 0$

$V_A = 20\ tf$

$\Sigma M_A = 0 \rightarrow 4 \cdot 4 + 20 \cdot 3 - 16 - 2 \cdot 6 - V_B \cdot 4 = 0$

$V_B = \frac{48}{4} = 12\ tf$

$H_A = 10\ tf$